The 30°-60° Right Triangle

SKETCH AND INVESTIGATE

Q1 $\angle CDB$ measures 90° because the median of an equilateral triangle is also an altitude. $\angle DBC$ measures 60° because it is an angle in an equilateral triangle. $\angle BCD$ measures 30° because it is half of an angle in an equilateral triangle.

Q2 In a 30°-60° right triangle, the hypotenuse is twice as long as the shortest leg.

Q3 The ratio of the area of the largest square to the area of the smallest square is 4:1. The areas have a ratio of 4:1 because the sides have a ratio of 2:1 and $2^2 : 1^2 = 4:1$.

Q4 a. If the smallest square has area x^2, the largest square will have area $4x^2$.

     b. If A represents the area of the square on the long leg, the Pythagorean theorem gives us $x^2 + A = 4x^2$. Subtracting x^2 from both sides gives us $A = 3x^2$.

     c. The ratio of the area of the square on the long leg to the area of the square on the short leg is 3:1. (If you can, check to make sure students verify this in their sketch.)

Q5 a. If the short leg has length x, the hypotenuse has length $2x$.

     b. The long leg would have length $x\sqrt{3}$ because the leg is the side of the square with area $3x^2$.

     c. The ratio $x\sqrt{3} : x$ simplifies to $\sqrt{3} : 1$.

Q6 1.713 (Answers will vary depending on the Precision setting for calculations. Change this in Preferences in the Edit menu.)

EXPLORE MORE

13. To make a Hide/Show button, select the object you want to show and hide; then choose **Edit | Action Buttons | Hide/Show**. Double-click any button with the **Text** tool to change its label. A student who finishes the activity early might make this sketch. Then you can use it with the whole class to practice calculating the missing sides for different 30°-60° right triangles.

The 30°-60° Right Triangle

The 30°-60° right triangle—formed by taking half of an equilateral triangle—has special relationships among its side lengths. These relationships make it easy to find all the side lengths if you know just one. In this activity you'll discover these relationships and why they hold.

SKETCH AND INVESTIGATE

You can use the tool **3/Triangle (By Edge)** from the sketch **Polygons.gsp**. If you need to relabel the triangle, use the **Text** tool. Click a point to show its label. Double-click a label to change it.
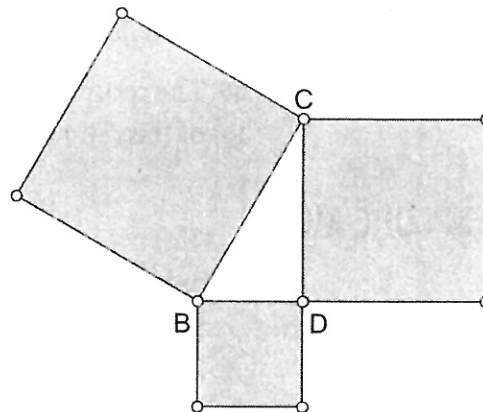
1. In a new sketch, construct an equilateral triangle ABC. Use a custom tool or construct it from scratch. Drag vertices to confirm that the construction is correct.

2. Hide the triangle's interior if necessary.

3. Construct the midpoint D of $\overline{AB}$.

4. Construct median CD.

Select the segments; then, in the Display menu, choose **Line Style | Dashed.**
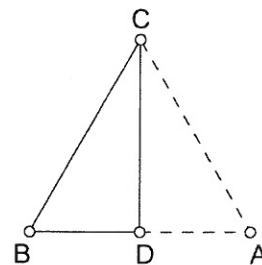
5. Change the line styles of $\overline{AB}$ and $\overline{AC}$ to dashed.

6. Construct $\overline{BD}$ and make its line style thin.

Q1 Without measuring, state the measure of each angle in $\triangle CDB$. For each angle, explain how you know it has that measure.

Q2 In a 30°-60° right triangle, how does the length of the hypotenuse compare to the length of the short leg? Answer without measuring.

7. Hide point A, $\overline{AB}$, and $\overline{AC}$.

8. Use a custom tool to construct squares on the three sides of the triangle as shown at right. Drag to make sure the squares are properly attached.

9. Measure the areas of the three squares.

Choose **Calculate** from the Number menu to open the Calculator. Click a measurement to enter it into a calculation.

10. Calculate the ratio of the largest area to the smallest area

11. Drag point B and observe this ratio.

Q3 What is this area ratio? Explain why this ratio is what it is. In your explanation, use what you know about the side lengths.

Q4 Now you'll use your answer to Q3 about the square on the hypotenuse and the square on the short leg to help you find the area of the square on the long leg. Answer the following questions.

   a. Suppose the smallest square has area x^2. What would be the area of the square on the hypotenuse? _____

   b. Use the Pythagorean theorem to find the area of the square on the long leg. Show your work.

   c. State the ratio of the area of the square on the long leg to the area of the square on the short leg. Calculate this ratio in your sketch and drag point B to confirm that this ratio applies to all 30°-60° right triangles. _____

Q5 Suppose the short leg had length x.

   a. What would be the length of the hypotenuse? _____

   b. What would be the length of the long leg? _____

   c. What would be the ratio of the length of the long leg to the length of the short leg? (State your answer in radical form.) _____

Select the hypotenuse and the short leg. Then, in the Measure menu, choose **Ratio.** Measure the other ratio in the same way.

12. To confirm your answers to Q5, measure the ratio of the hypotenuse length to the short leg length. Also measure the ratio of the long leg length to the short leg length. Drag point B to confirm that these ratios apply to all 30°-60° right triangles.

Q6 The second ratio you measured in step 12 should be the decimal approximation of the ratio you wrote in Q5c. Record this decimal approximation. _____

EXPLORE MORE

13. To test how well you can apply your discoveries, make a Hide/Show action button for each side length measurement. Show one side length and hide the other two. Then calculate the two hidden lengths. Show the hidden lengths to check your calculations. Try this several times, changing the triangle each time and showing different side lengths. Repeat until you think you can calculate the missing side lengths correctly every time.