

Objective: Students use an animated Sketchpad model for adding integers on the number line. They investigate addition of two positive numbers, addition of two negative numbers, and addition of a positive and a negative number.

Student Audience: Pre-algebra/Algebra 1

Prerequisites: None. This will be review for most Algebra 1 students.

Sketchpad Level: Easy. Students manipulate a prepared sketch.

Activity Time: 20–30 minutes. You may want to combine this activity and the Subtracting Integers activity in a single class period.

Setting: Paired/Individual Activity (use **Adding Integers.gsp**) or Whole-Class Presentation (use **Adding Integers Present.gsp**)

Use this activity as an introduction to integer addition for pre-algebra students, as a start-of-the-year refresher for Algebra 1 students, or as a supplemental activity for any student having difficulty with the topic. It's important for students to have a mental image of operations on integers. Even strong students who rely on verbal rules make careless mistakes that could be avoided by having an internalized picture.

The picture of addition presented here is a geometric model in which each number is represented by a vector. (The activity calls them *arrows* because students may not be familiar with the term *vector*.) Vectors incorporate both magnitude and direction (representing the absolute value and the sign of the integer), so practice with this model helps students understand how the signs of the addends come into play.

This activity contains lots of questions for students, who develop their understanding through the process of manipulating the sketch and describing what they observe. Encourage them to write clear and detailed explanations (and to use complete sentences) when they answer the questions; the extra time it takes them to do so is well spent.

If there's time and you have a presentation computer with a projector, have different students use Sketchpad to demonstrate to the class their observations or the problems they made up. It's a big help to students if they can listen to, evaluate, and discuss the descriptions and conclusions of their classmates.

INVESTIGATE

Students may be unfamiliar with *model* as a transitive verb; consider reviewing with them the various uses of this word.

- Q1** In their final positions, the second arrow starts from where the first arrow ends, and the answer (13) is at the end of the second arrow. Encourage students to be detailed and specific in their answer to this question.
- Q2** Answers will vary but should include only positive numbers.
- Q3** Each lower arrow is exactly the same size and direction as the corresponding upper arrow.
- Q4** The sum of $-6 + (-3)$ is -9 .
- Q5** Answers will vary but should include only negative numbers.
- Q6** Whether adding two negative or two positive numbers, both arrows go the same way, taking the sum farther away from the center of the number line (farther away from zero). The difference is that the arrows go to the right when the numbers are positive but go to the left when they're negative.
- Q7** When you add two negative numbers, you cannot get a positive sum. Both numbers take the sum in the negative direction from zero, so the sum must be negative.
- Q8** As students model various problems, walk around the room and observe them to make sure they can model any problem they are given.

$$\begin{array}{ll} 7 + (-4) = 3 & -4 + 7 = 3 \\ -6 + 2 = -4 & 2 + (-6) = -4 \\ -3 + 7 = 4 & 3 + (-7) = -4 \\ 2 + (-5) = -3 & -2 + 5 = 3 \end{array}$$

Q9 When you add a positive and a negative integer, the number that has the larger absolute value tells you whether the answer will be positive or negative. In other words, the sign of the result is the same as the sign of the longer arrow.

EXPLORE MORE

Q10 Each student will model different problems. In every case, the two numbers must be opposites, so that their arrows are the same length but point in opposite directions.

Q11 The order does not matter when you add two numbers. The arrows determine how far you go and in which direction, and it doesn't matter if you follow the first arrow and then the second, or if you follow the second arrow and then the first.

WHOLE-CLASS PRESENTATION

Start the whole-class presentation by animating the addition of two positive integers (Q1–Q3 of the activity). Open the sketch **Adding Integers Present.gsp** and press the step-by-step buttons one at a time, pausing between animations. Ask students to describe what they see as the animation progresses, and be sure to get observations from

several different students. Press the *Reset* button, change the problem by dragging both circles (while leaving the numbers positive), and press the step-by-step buttons again.

Next animate the addition of two negative numbers (Q4–Q7 of the activity). Press *Reset*, drag the numbers so they are both negative, and ask students to predict what will happen now. Use the step-by-step buttons to test their conjectures. Without resetting, ask questions Q6 and Q7, and experiment by dragging to change the values of the numbers.

When students are satisfied with the results of adding two negative numbers, animate the addition of numbers with different signs. Reset again and drag the circles so one of the numbers is positive and one is negative. Ask students to predict how the arrows will behave. (Try to get students to concentrate on the behavior of the model rather than on the numeric answer.) Use the buttons again to show the behavior. Model several more problems (such as those in Q8) involving a positive and a negative number.

Finish the class discussion using Q9, Q10, and Q11. When students propose an answer to one of these questions, have them manipulate the sketch to show why their answer makes sense.