

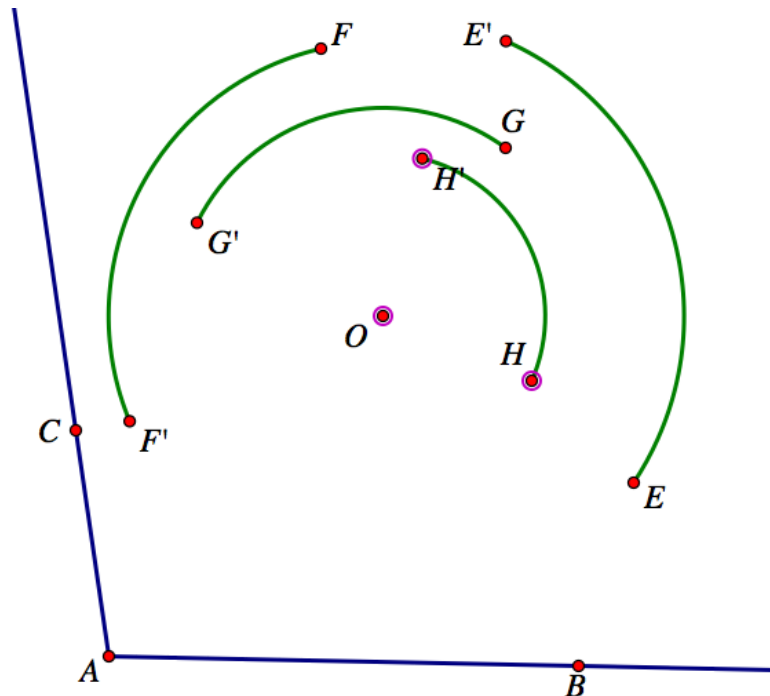
# Rotation and Arcs of Circles <sup>1</sup>

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Turning and changing direction is an important part of daily experience. You turn a corner, turn around, or turn your bicycle. You turn a steering wheel to change a car's direction. In this section you will examine how turning and the geometry of arcs are related.

## SKETCH AND INVESTIGATE

1. Construct rays  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$  to form  $\angle BAC$ .
2. Select points  $B, A$ , and  $C$  (in that order) and choose Mark Angle from the Transform menu.
3. Place a point not on  $\angle BAC$  and label the point  $O$ .
4. Select point  $O$  and choose Mark Center from the Transform menu.
5. Place four additional points  $E, F, G$ , and  $H$  in the sketch. Select these four points and choose Rotate from the Transform menu. Four new points,  $E', F', G'$ , and  $H'$  will appear.
6. Construct an arc by selecting center point  $O, E$ , and  $E'$  (in that order). Choose Arc on Circle from the Construct menu. Repeat this to create arcs  $FF', EE', GG'$ , and  $HH'$ .



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<sup>1</sup> Adapted from King, J. (1996). *Geometry through the circle*. Key Curriculum Press: Berkeley, CA.

Q1. Drag point  $C$  to change the angle of rotation and notice how the arcs change. Explain what is happening.

7. Find the angle measures of the arcs. Select the arcs and choose Arc Angles from the Measure menu.

Q2. What do you notice about the arc angles? Drag point  $C$  and explain what changes.

8. Find the lengths of the arcs. Select the arcs and choose Arc Length from the Measure menu.

Q3. Notice which arcs have the greatest arc length.

9. Measure the distances from  $O$  to each of the points  $E, F, G, H$ .

Q4. Drag point  $E$  and explain how the arc length changes.

10. Drag point  $E$  so that arc  $EE'$  is approximately twice the length of arc  $HH'$ .

Q5. What do you notice about the distances these arcs are from point  $O$ ?

## Rotations and Arcs of Circles (continued)

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Continue with the same sketch.

### SKETCH AND INVESTIGATE

1. Construct segments that connect point  $O$  to each of the points  $E, E', F, F', G, G', H$ , and  $H'$ .
2. Drag points  $E, F, G$ , and  $H$  so that they are different distances from  $O$  but all four points are approximately on a line through point  $O$ .

Q1. Explain how points  $E', F', G'$ , and  $H'$  and point  $O$  are related in this figure.

3. Measure angle  $EOE'$ .

Q2. Explain how the measure of angle  $EOE'$  is related to the measure of angle  $ABC$  and the measure of the arc angles.

What does it mean for two figures to be similar if the figures are not triangles? For one thing, corresponding ratios of distances should be the same.

4. Use the Calculate command from the Number menu to divide the length of an arc

$$\frac{\text{Length } \widehat{EE'}}{OE} = 0.82$$

by the distance the arc is from the center.  
Repeat this for another arc.

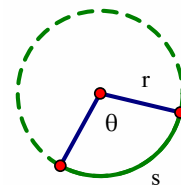
Q3. What do you notice about the two ratios?

5. To determine how this value is related to other values in the sketch, choose Preferences from the Edit menu. Change the angle measure to radians.

Q4. Explain how the quotient is related to other measures in the sketch.

Q5. Explain how what you noticed in the sketch could be used to develop a formula for finding the arc length ( $s$ ) given the angle measure ( $\theta$ ) of a sector and the radius ( $r$ ) of a circle.

Q6. Explain how the arc length is related to the angle  $\theta$  and the Circumference of the circle.



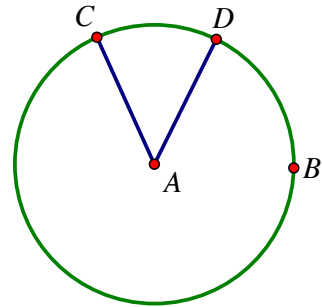
# Sectors of Circles

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Areas of sectors of circles are especially important to consider when creating or interpreting pie graphs as well as solving problems. In this activity you will develop a formula for calculating the area of a sector of a circle.

## SKETCH AND INVESTIGATE

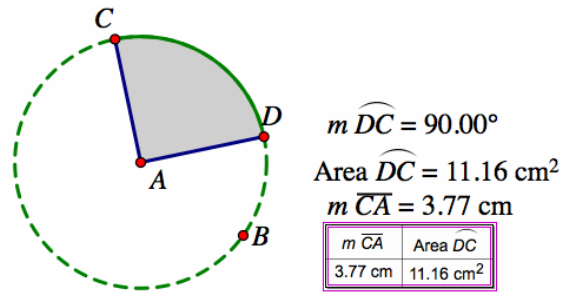
1. From the Edit menu, choose Preferences and make sure the unit for angle measures is degrees rather than radians.
2. Use the Compass Tool to create a circle. Label the center  $A$  and radius point  $B$ .
3. Select the circle and choose Point on Circle from the Construct menu. Do this again. Label the two points on the circle  $C$  and  $D$ .
4. Construct segments  $\overline{AC}$  and  $\overline{AD}$ .
5. Select point  $A$ , point  $C$ , and point  $D$ . Choose Arc on Circle from the Construct menu.
6. With the arc highlighted choose Arc Angle from the Measure menu.
7. With the arc highlighted, choose Arc Interior  $\rightarrow$  Arc Sector from the Construct menu.
8. Select the interior of the sector and choose Area from the Measure menu.
9. Drag point  $C$  until the measure of arc  $CD$  is approximately 90 degrees.



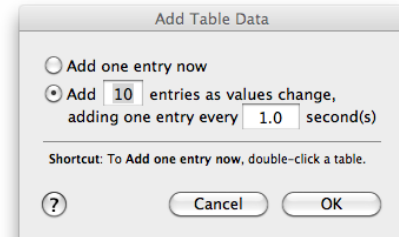
Q1. Drag point  $B$  to adjust the radius so that it becomes shorter and longer. What other measures change? What other measures stay the same? Explain.

10. Select  $\overline{AC}$  and choose length from the Measure menu.
11. Select the measure of the radius and then select the area of the sector. Choose Tabulate from the Number menu.

12. With the table highlighted, chose Add Table Data from the Number menu. Select Add 10 entries as values change, adding one entry every 1 second. Press OK.



13. Drag point B to make the circle larger.
14. Select the table then choose Plot Table Data from the Graph menu.
15. From the Graph menu choose Grid Form  $\rightarrow$  Rectangular Grid. Adjust the scales on the x and y axis to view all of the plotted points.



The plotted points appear to lie along a curve that could be described using a quadratic function. Since the graph passes through the point (0,0) the graph will be of the form  $y = a \cdot x^2$ . We can create a new parameter to represent the value of  $a$  and change the value of  $a$  until the graph passes through the plotted points.

16. Select New Parameter from the Number menu. Change the name of the parameter to  $a$ .
17. Select Plot New Function from the Graph menu. Click on the parameter  $a$  press  $*$  and type  $x^2$ .
18. Change the value of  $a$  until the graph passes through the plotted points.
- Q2. Write a function rule to describe the area of a sector as a function of the length of the radius.
19. To determine how the value of  $a$  is related to the circle, choose Calculate from the Number menu and divide  $a$  by  $\pi$ .
- Q3. How is  $\frac{a}{\pi}$  related to the 90 degree arc angle?
20. Change the measure of the arc angle to 180. Select the length of the radius and then the area of the sector.
21. Choose Plot as (x,y) from the Graph menu.

22. Choose Trace Plotted Point from the Display menu. Change the radius and observe the trace. Modify the value of  $a$  so that the graph passes through the trace.

23. Choose Calculate from the Number menu and divide  $a$  by pi. How is  $\frac{a}{\pi}$  related to the 180 degree arc angle?

Q4. Create a formula to find the area of the sector of a circle if you are given the measure of a central angle and the length of the radius or the circle.