

Concepts

Function explored: parabolic.

Real-world concepts such as free-falling and bouncing objects, gravity, and constant acceleration are examples of parabolic functions. This activity investigates the values of height, time, and the coefficient A in the quadratic equation, $Y = A(X - H)^2 + K$, which describes the behavior of a bouncing ball.

Materials

- ✓ calculator
- ✓ CBR
- ✓ calculator-to-CBR cable
- ✓ large (9-inch) playground ball
- ✓ TI ViewScreen (optional)

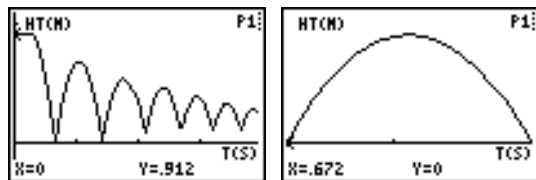
Hints

This activity is best performed with two students, one to hold the ball and the other to push **TRIGGER**.

See pages 6–12 for hints on effective data collection.

The plot should look like a bouncing ball. If it does not, repeat the sample, ensuring that the CBR is aimed squarely at the ball. A large ball is recommended.

Typical plots



Explorations

After an object is released, it is acted upon only by gravity (neglecting air resistance). So A depends on the acceleration due to gravity, -9.8 meters/second² (-32 feet/second²). The negative sign indicates that the acceleration is downward.

The value for A is approximately one-half the acceleration due to gravity, or -4.9 meters/second² (-16 feet/second²).

Typical answers

1. time (from start of sample); seconds; height / distance of the ball above the floor; meters or feet
2. initial height of the ball above the floor (the peaks represent the maximum height of each bounce); the floor is represented by $y = 0$.

3. The Distance-Time plot for this activity does not represent the distance from the CBR to the ball. BALL BOUNCE flips the distance data so the plot better matches students' perceptions of the ball's behavior. $y = 0$ on the plot is actually the point at which the ball is farthest from the CBR, when the ball hits the floor.
4. Students should realize that the x-axis represents time, not horizontal distance.
7. The graph for $A = 1$ is both inverted and broader than the plot.
8. $A < -1$
9. parabola concave up; concave down; linear
12. same; mathematically, the coefficient A represents the extent of curvature of the parabola; physically, A depends upon the acceleration due to gravity, which remains constant through all the bounces.

Advanced explorations

The rebound height of the ball (maximum height for a given bounce) is approximated by:

$$y = hp^x, \text{ where}$$

- y is the rebound height
- h is the height from which the ball is released
- p is a constant that depends on physical characteristics of the ball and the floor surface
- x is the bounce number

For a given ball and initial height, the rebound height decreases exponentially for each successive bounce. When $x = 0$, $y = h$, so the y-intercept represents the initial release height.

Ambitious students can find the coefficients in this equation using the collected data. Repeat the activity for different initial heights or with a different ball or floor surface.

After manually fitting the curve, students can use regression analysis to find the function that best models the data. Select a single bounce using PLOT TOOLS, SELECT DOMAIN. Then QUIT from the MAIN MENU. Follow the calculator operating procedures to perform a quadratic regression on lists L1 and L2.

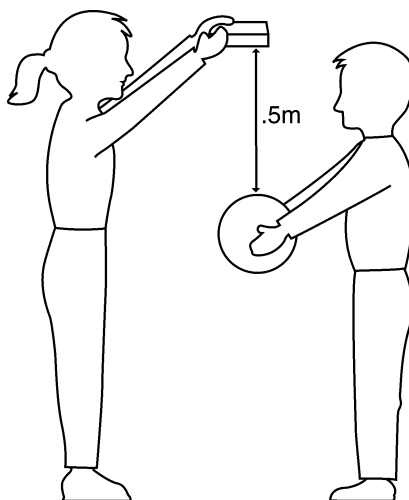
Extensions

Integrate under Velocity-Time plot, giving the displacement (net distance traveled) for any chosen time interval. Note the displacement is zero for any full bounce (ball starts and finishes on floor).

Data collection

- 1 Begin with a test bounce. Drop the ball (do not throw it).

Hints: Position the CBR at least 0.5 meters (1.5 feet) above the height of the highest bounce. Hold the sensor directly over the ball and make sure that there is nothing in the clear zone (see page 7).



- 2 Run the RANGER program (see page 5 for keystrokes for each calculator).
- 3 From the MAIN MENU choose APPLICATIONS. Choose METERS or FEET.
- 4 From the APPLICATIONS menu choose BALL BOUNCE. General instructions are displayed. BALL BOUNCE automatically takes care of the settings.
- 5 Hold the ball with arms extended. Press **ENTER**. The RANGER program is now in Trigger mode. At this point, you may detach CBR from the calculator.
- 6 Press **TRIGGER**. When the green light begins flashing, release the ball, and then step back. (If the ball bounces to the side, move to keep the CBR directly above the ball, but be careful **not** to change the height of the CBR.)

You can hear a clicking sound as the data is collected. Data is collected for time and distance, and calculated for velocity and acceleration. If you have detached the CBR, reattach it when data collection is finished.

- 7 Press **ENTER**. (If the plot doesn't look good, repeat the sample.) Study the plot. **Answer questions 1 and 2 on the activity sheet.**
- 8 Observe that BALL BOUNCE automatically flipped the distance data. **Answer questions 3 and 4.**

Explorations

The Distance-Time plot of the bounce forms a parabola.

- ❶ Press **[ENTER]**. From the PLOT MENU, choose PLOT TOOLS, and then SELECT DOMAIN. We want to select the first full bounce. Move the cursor to the base of the beginning of the bounce, and press **[ENTER]**. Move the cursor to the base at the end of that bounce, and then press **[ENTER]**. The plot is redrawn, focusing on a single bounce.
- ❷ The plot is in TRACE mode. Determine the vertex of the bounce. *Answer question 5* on the activity sheet.
- ❸ Press **[ENTER]** to return to the PLOT MENU. Choose MAIN MENU. Choose QUIT.
- ❹ The *vertex form* of the quadratic equation, $Y = A(X - H)^2 + K$, is appropriate for this analysis. Press **[Y=]**. In the Y= editor, turn off any functions that are selected. Enter the vertex form of the quadratic equation: $Yn=A*(X-H)^2+K$.
- ❺ On the Home screen, store the value you recorded in question 5 for the height in variable **K**; store the corresponding time in variable **H**; store 1 in variable **A**.
- ❻ Press **[GRAPH]** to display the graph. *Answer questions 6 and 7.*
- ❼ Try $A = 2, 0, -1$. *Complete the first part of the chart in question 8 and answer question 9.*
- ❽ Choose values of your own for A until you have a good match for the plot. *Record your choices for A in the chart in question 8.*
- ❾ Repeat the activity, but this time choose the last (right-most) full bounce. *Answer questions 10, 11, and 12.*

Advanced explorations

- ❶ Repeat the data collection, but do not choose a single parabola.
- ❷ Record the time and height for each successive bounce.
- ❸ Determine the ratio between the heights for each successive bounce.
- ❹ Explain the significance, if any, of this ratio.

Activity 4—Bouncing ball

Name _____

Data collection

1. What physical property is represented along the x-axis? _____
What are the units? _____
What physical property is represented along the y-axis? _____
What are the units? _____
2. What does the highest point on the plot represent? _____
The lowest point? _____
3. Why did the BALL BOUNCE program flip the plot? _____
4. Why does the plot look like the ball bounced across the floor? _____

Explorations

5. Record the maximum height and corresponding time for the first full bounce. _____
6. Did the graph for $A = 1$ match your plot? _____
7. Why or why not? _____
8. Complete the chart below.

A	How do the data plot and the Yn graph compare?
1	
2	
0	
-1	

9. What does a positive value for A imply? _____
What does a negative value for A imply? _____
What does a zero value for A imply? _____
10. Record the maximum height and corresponding time for the last full bounce. _____
11. Do you think A will be bigger or smaller for the last bounce? _____
12. How did A compare? _____
What do you think A might represent? _____