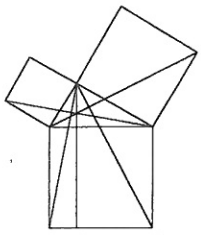
Euclid's Proof, a.k.a. Pythagoras' Pants

Name(s): _____

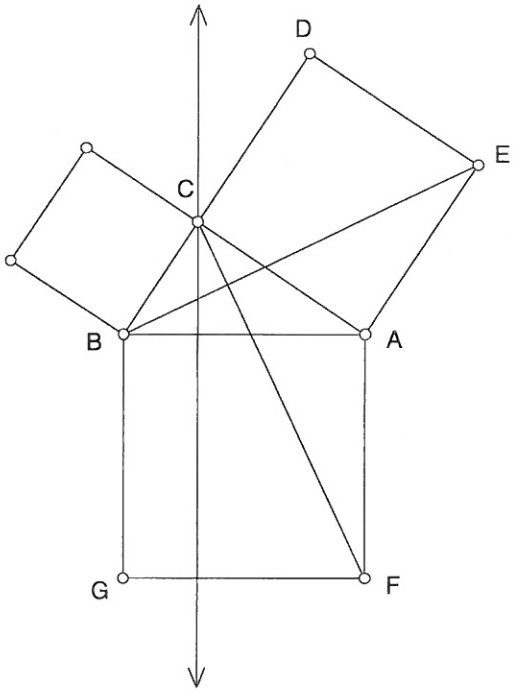


Russian students less reverent than Arab scholars have called the figure in Euclid's proof Pythagoras' pants.

The Pythagorean theorem is one of the important milestones in Euclid's *Elements*. This work, written around 300 B.C.E., has had a tremendous influence on mathematics because of the systematic way in which it presents geometry propositions logically derived from one another. Euclid arrives at the Pythagorean theorem and its converse as the 47th and 48th (and final) propositions of Book 1 (out of 13). It's thanks to Arab scholars and Moorish scholars of northern Africa and southern Spain that much of ancient Greek mathematics survived. The figure on page 7 is from a manuscript of Tâbit ibn Qorra's translation of Euclid. Arab scholars referred to the figure as "the figure of the bride."

Sketch and Investigate

1. Construct a right triangle and squares on the sides, as shown.
2. Construct a line through the right angle vertex, perpendicular to the hypotenuse.
3. Construct $\overline{CF}$ and $\overline{BE}$.



Euclid's proof is pretty quick, if you can establish the relationships between the triangles and the squares. But those relationships can be subtle in the complex diagram. A little further construction can add a dynamic element to your sketch that will give you more insight into how the proof works.

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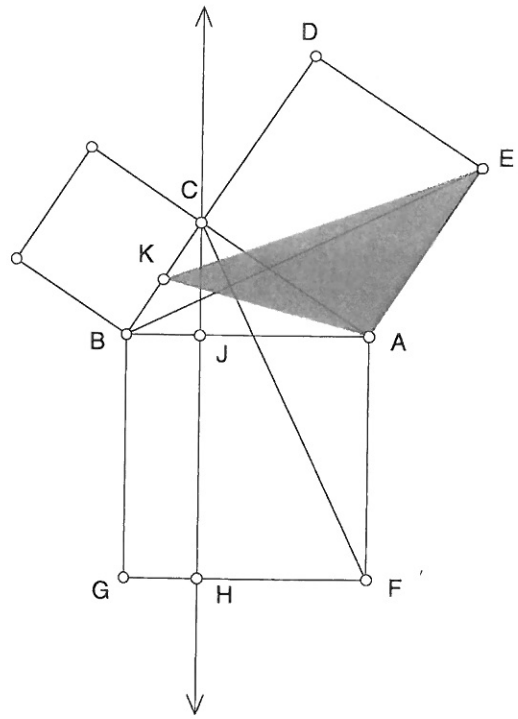
4. Construct a point K on side BC. Construct the polygon interior EAK. You should be able to drag point K back and forth along $\overline{BC}$.



Q1 When point K coincides with point C, what can you say about the relationship between the shaded triangle and square $ACDE$?

Q2 Drag point K toward point B. Does the area of the triangle change? Explain why this is so. *Hint:* If you consider AE as the length of the base, what's the height of ΔEAK? Does that change?

5. Drag point K until it coincides with point B. Now mark A as center, select the triangle interior, and rotate it by 90°.

Q3 What new triangle interior do you get? How are triangles EAB and CAF related?

Q4 In questions Q1 and Q2, you established a relationship between ΔEAB and square $EACD$. How are ΔCAF and rectangle $JAFH$ related?

Prove

The investigation gives you a start on Euclid's proof. First, show $\Delta EAB \cong \Delta CAF$. Then supply the necessary steps to show the area of square $ACDE$ is equal to the area of rectangle $AJHF$. A similar argument will prove the area of the small square is equal to the area of rectangle $BGHJ$.

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