Exponential Relationships—Population Growth

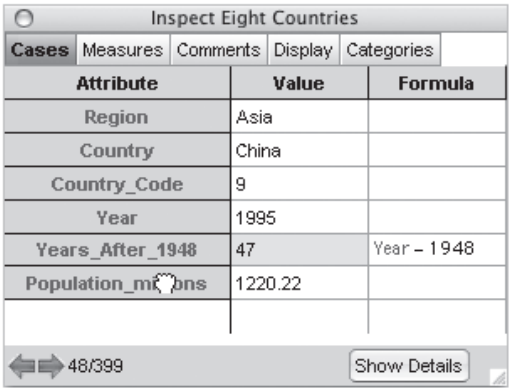
Governments concern themselves with providing their populations with necessities, such as safe water and food, housing, and medical care. To plan for future needs, they predict their nation's population sizes. Three countries have hired you to help them predict their population sizes in the year 2020.

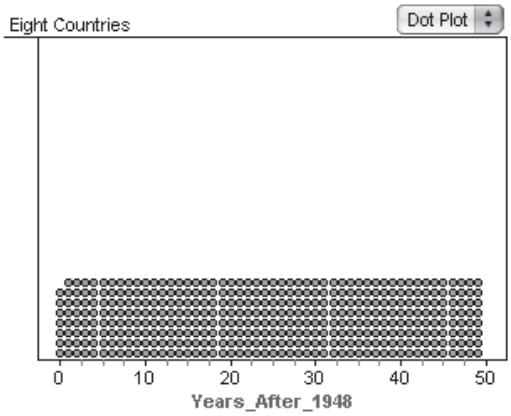
Q1 What types of things cause a population to change size? Make a list of important factors, beginning with the most influential.

INVESTIGATE

Drag a graph from the shelf, double-click on the collection (but not its name) to open the inspector, and drag attributes from the Eight Countries inspector to the axes of the graph.

1. The first country to hire you is Kenya. Open **Population.ftm** and create a scatter plot of (*Years_After_1948, Population_millions*).





After adding *Population_millions* to the vertical axis, select the graph and go to **Object | Add Filter.** Type Country = "Kenya." Click **OK.** Reselecting **Scatter Plot** as the graph type at the top right will automatically adjust the vertical scale.

2. Add a filter for Country = "Kenya."

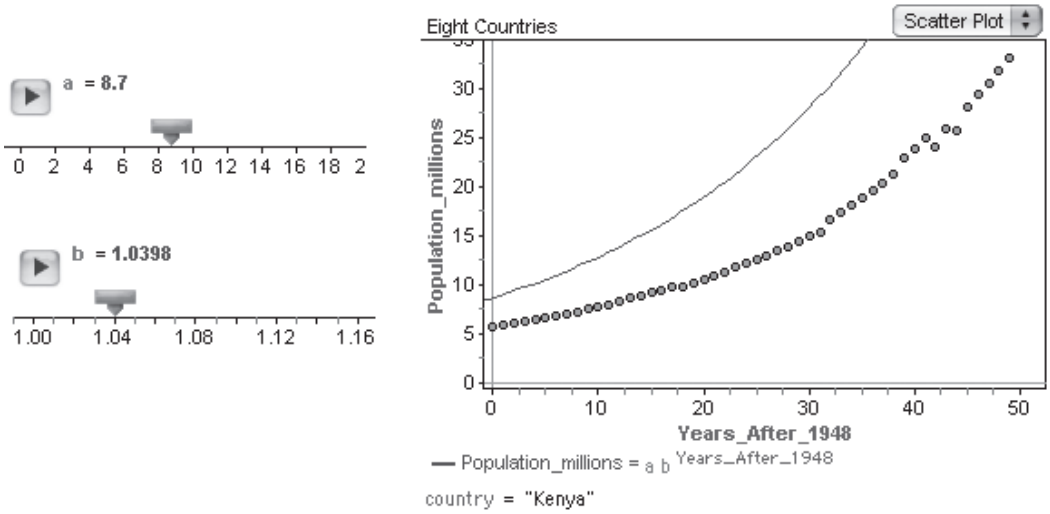
The major influence on next year's population is the number of people living in the country today. A good mathematical model for next year's population in many countries involves multiplying the current population by a constant number that represents birth, death, immigration, and emigration. Because of repeated multiplication by this constant number, say b, this model for population growth is exponential and looks like $y = ab^x$. To predict the growth of Kenya's population, you will need to find appropriate values of a, the current population, and b, the rate of growth.

Population is measured in millions as indicated in the attribute name, so we don't include units on the *a* slider.

3. Drag down two sliders and give them names such as a and b (with no units).

Select the graph of Kenya's population and go to **Graph | Plot Function.** Enter a • b ^ Years_After_1948 and click **OK.**

4. Use the values of the sliders to add the graph $ab^{Years\_After\_1948}$ to your scatter plot.



5. Adjust both the *a* and *b* sliders until the graph best matches the data. You will probably want the endpoints of slider *b* to be quite close to 1.

Q2 How does changing the sliders affect the shape of the graph of $Population\_millions = ab^{Years\_After\_1948}$?

Q3 Write the formula for *Population_millions* using your best values for *a* and *b*. What do these values tell you about the population of Kenya?

You may first want to calculate how many years 2020 is after 1948.

Q4 Using these values, what do you predict Kenya's population will be in 2020?

6. The next country to hire you is Israel. Double-click on the filter Country="Kenya" and change "Kenya" to "Israel."

7. Adjust the sliders to match the new scatter plot of Israel's population as well as you can.

Q5 Describe how this graph compares with the graph for Kenya. Use the graph to predict Israel's population in 2020.

8. Because the exponential graph seems to be growing faster than Israel's population, a linear model might be better than an exponential model for this country. Add a movable line to the graph to decide whether this is a better model.

Q6 Where on the graph is the value of *a* represented in each model?

$$Population\_millions = ab^{Years\_After\_1948}$$

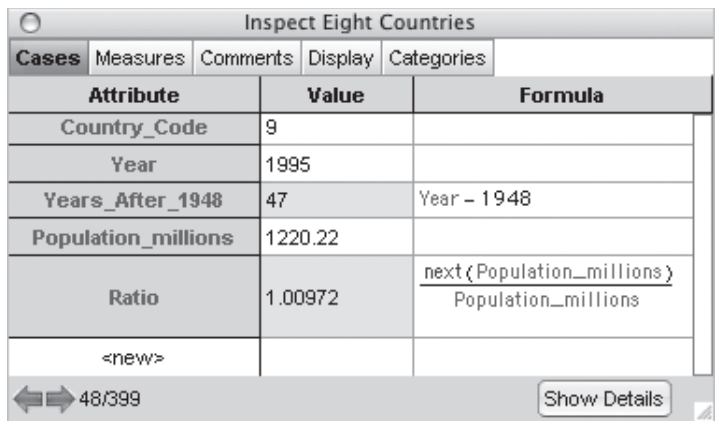
$$Population\_millions = a + b \cdot Years\_After\_1948$$

Q7 Give your best model and your prediction for the 2020 population of Israel.

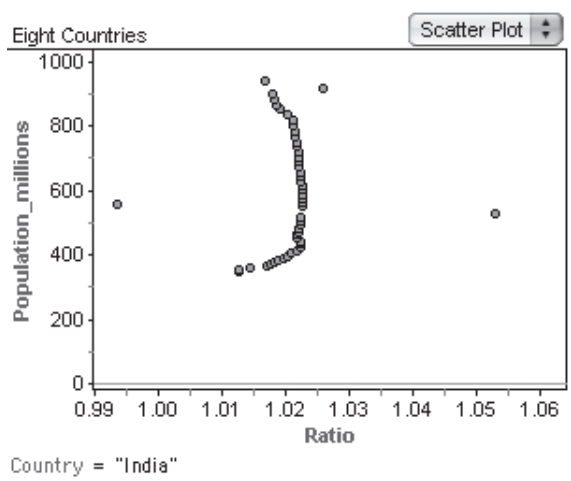
9. The third country to hire you is India. Repeat the process to find a model for India's population.

Q8 What do you predict will be India's population in 2020? Justify your answer.

10. You sometimes need to come up with exponential population models on the fly, without the benefit of Fathom. You can use Fathom to see how to do this. The value of *a* is the initial population. To find the value of *b*, you can use the ratio of the population one year to the population of the previous year. Open the inspector for the collection and enter a new attribute, perhaps called *Ratio,* with the formula next(Population_millions)/Population_millions.



11. To see these values, make a new graph with *Population_millions* versus *Ratio.*



Q9 What can you learn from the fact that these ratios are mostly in a vertical line?

Q10 To see the value more exactly, you might plot the value of median(*Ratio*). How can you use ratios to calculate the value of *b*?

Q11 Carefully explain the role of *a* and *b* in the graph of

$$Population\_millions = ab^{Years\_After\_1948}$$

EXPLORE MORE

1. Use the graphs that you created to determine what happens to the graph of $Population\_millions = ab^{Years\_After\_1948}$ if a or b are negative, if b is 1, or if b is positive but less than 1. Do your observations change how you described the role of a and b in Q11?

2. Suppose the average salary for teachers in one American state is $30,000 and is growing at 4% per year, whereas in a second state, the average salary is more, $40,000, but is growing at a lower annual rate of 2%. Write an equation for each state's average salary. Use Fathom to see when the first state's amount catches up to the second state's.