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Geometry

The Circumference Function

What is the relationship between a circle's circumference and its diameter? In this activity, you'll discover (or, maybe, rediscover) this important relationship.

MAKE A CONJECTURE

You probably already have some ideas about what will happen. The relationship might even be familiar to you. Think about what you expect to see.

1. Take one of your circular objects. Imagine measuring its diameter and circumference and comparing the two quantities. Think about ways to use words, numbers, or algebra to describe the relationship as generally as possible.

Q1 What do you think the relationship between diameter and circumference is for any circle?

2. Now imagine measuring the diameter and circumference of many circles and making a graph of points in the form (*Diameter, Circumference*).

Q2 Sketch the way you think the graph of *Circumference* versus *Diameter* will look. Be sure to scale and label your axes.

EXPERIMENT

Now you'll actually measure the diameters and circumferences of your circular objects. Use centimeters for all measurements. Record your results in a table.

3. Measure each diameter by holding your ruler or measuring tape across the widest part of the circle.

4. Measure each circumference by wrapping your measuring tape or string around the outside of the circle. If you use string, unwrap the string and then lay it along your ruler to get the measurement.

**Q3** How do your measurements compare to your conjecture in Q1?

INVESTIGATE

Next you'll put the data into Fathom and learn ways to analyze it.

5. Start Fathom. You'll have an empty document.

6. Choose **New | Case Table** from the **Object** menu. An empty case table appears.

7. Click the word <**new**> at the top of the column in the table. Type `Diameter` and press **Enter** to make a new attribute. Do the same to create an attribute for *Circumference.*

8. Enter your measurements in the case table. Type the first value and its unit under each attribute and the unit will be applied to every value under the attribute.

	Diameter	Circumference
units	centimeters	centimeters
1	4.0 cm	12.4 cm
2	8.0 cm	25.2 cm

When two quantities are related, a graph of points is a good way to see the relationship. You'll make a graph called a *scatter plot.*
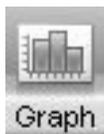
Graph

9. Drag a new graph from the object shelf into your document. An empty graph appears.

10. Drag *Diameter* from the case table to the graph. Drop the attribute below the horizontal axis. Then drag *Circumference* to the vertical axis.

Q4 Sketch your scatter plot, with scaled and labeled axes. How does it compare to your conjecture in Q2?

Q5 Does there appear to be a pattern that is true for all circles? What type of continuous graph would go through the points in your scatter plot?

Q6 Would the continuous graph go through the point (0, 0)? Should it? Explain.
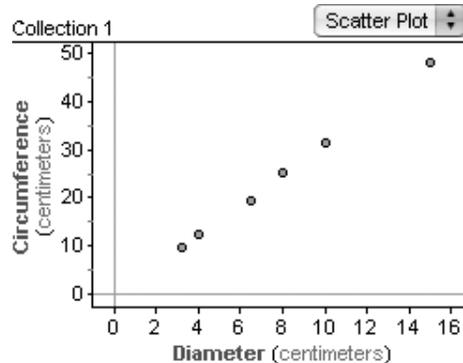
You probably noticed a linear pattern in Q5. Fathom gives you several tools for making a line that fits your data. You'll use the simplest of them, a movable line.

11. Select the graph by clicking on it. Choose **Add Movable Line** from the **Graph** menu. A line and its equation appear. Choose **Lock Intercept at Zero** from the **Graph** menu to make the line pass through the origin.

12. Get comfortable with changing the line. Then move the line to best fit the points. (*Hint:* When you try to move the line, you may accidentally grab a data point and change your data. If this happens, choose **Undo** from the **Edit** menu until everything is all right.)

Q7 Add the line to your sketch of the scatter plot. What is the equation of your line? What number in the equation represents the relationship between the diameter and the circumference? How is this number represented by the graph of the line?

Q8 Your line indicates a linear relationship. But your points probably aren't all on the line, which means your measurements had some variability. Why would this be? Do you think the relationship between circumference and diameter is really linear? Explain.

The linear equation that you found indicates that the *ratio* of circumference to diameter remains the same. Calculating this ratio for each circle is another way to look at the relationship between diameter and circumference.

13. In the case table, create a new attribute *CircDiam*. This attribute will be the ratio of circumference to diameter.

14. Select the *CircDiam* column and choose **Edit Formula** from the **Edit** menu. The formula editor opens. Type `Circumference/Diameter` and click **OK.** The ratios appear.
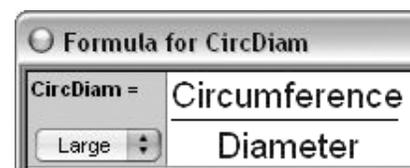
Q9 How do the ratios of circumference to diameter compare to the other work you have done in this activity? Does there seem to be an "average" ratio?

15. Drag a new summary table from the object shelf. Drop the attribute *CircDiam* in the row or column of the summary table. The mean is automatically calculated.

Q10 What is the mean value for the ratios?

Q11 Based on your work in Fathom, what is the relationship between diameter and circumference for any circle? Does this match your original conjecture from Q1? How well did your actual measurements conform to the relationship?

Q12 Did your work in Fathom reveal any surprises? Describe and explain.

EXPLORE MORE

1. In this activity, you used centimeters to measure both *Diameter* and *Circumference.* What happens if you measure everything in inches? What happens if you measure one attribute in centimeters and the other in inches? Make conjectures, collect revised data, and then analyze the data in Fathom.

2. Cut string into various lengths. For each string, measure the length to get an attribute *Length.* Using a simple knot, tie the ends of each string together to make a loop. Stretch each loop into a circle and measure the *Diameter.* Predict and then analyze the relationship between *Length* and *Diameter.* What relationship did you find? How is this relationship similar to or different from the relationship that you studied in the activity?

Area and Perimeter

How are the area and perimeter of a rectangle related? You probably know the formulas by heart:

$$Area = Length \cdot Width$$

$$Perimeter = 2(Length + Width)$$

But if you look at data for many different rectangles, would you notice any patterns? Are there relationships between area and perimeter? You'll use Fathom to explore these questions.

EXPERIMENT

The first step is to experiment with many different rectangles and gather data. You could get data by actually drawing or cutting out rectangles and measuring their lengths and widths, but that would get tiring. Instead, you'll use Fathom to generate random numbers for the lengths and widths. Using technology to randomly generate data for an experiment is one method of doing a *simulation*.

1. Open a new Fathom document and create a new case table.

2. Make two new attributes, *Length* and *Width.*

3. With the case table selected, choose **New Cases** from the **Collection** menu. Type `200` in the dialog box and click **OK.** The cases appear in your table, although they have no values.

For this activity, you'll assume that the length or width of a rectangle can be between 0 and 10 units.

4. Select the attribute *Length* and choose **Edit Formula** from the **Edit** menu. Type `random(10)` in the formula editor and click **OK.** The *Length* column fills with randomly generated numbers. If you want, choose **Rerandomize** from the **Collection** menu to see a different set of random numbers.

Q1 Look at the values of *Length.* What does the formula *random*(10) do? What would *random*(100) do? How about *random*()? (*Hint:* If you aren't sure, read about the *random* function in Fathom Help.)

5. Use the same formula to define *Width.*

Graph

6. Drag a new empty graph from the object shelf. Drag *Length* from the case table to the horizontal axis. You'll see a dot plot; you can change the graph to a histogram if you like. Make a dot plot or histogram of *Width,* too.
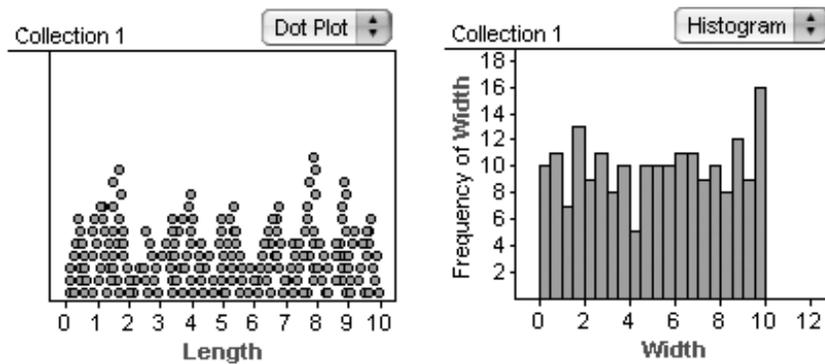
Q2 Describe the general "shape" of the graphs of *Length* and *Width.* If you want, rerandomize the data several times to get a better feel for the general shape. Why do these graphs have this shape?

7. Make another new graph. This time make a scatter plot with *Length* on one axis and *Width* on the other axis. Rerandomize the data a few times.

Q3 Describe any patterns in the scatter plot. Why does the scatter plot look the way it does?

Now that you have data that represent random rectangles, you need to find the perimeter and area of each. After all, you want to look for relationships between area and perimeter.

8. Make two new attributes, *Area* and *Perimeter.* Define each with a formula that uses *Length* and *Width.*

Q4 Copy on paper all of the values for the first two cases in your case table. Use by-hand calculations to verify that your formulas for *Area* and *Perimeter* are correct.

MAKE A CONJECTURE

So far, you've simulated many rectangles by randomly generating length and width, and you've calculated the area and perimeter. You've looked at graphs that help you see how length and width are related. Now, let's see how area and perimeter are related. But first, predict what you expect to find.

Q5 If you made a scatter plot of *Area* versus *Perimeter,* what do you think it would look like? Make a sketch of what you would expect to see or describe it thoroughly. Explain why you think it would look that way.

INVESTIGATE

9. In Fathom, actually make the scatter plot of *Area* versus *Perimeter*. Put *Perimeter* on the horizontal axis and *Area* on the vertical axis. You might want to rerandomize a few times to make sure you understand the general relationship.

Q6 Make a sketch of the actual graph or describe it thoroughly. Did the actual graph match your prediction? Is there anything about the graph that surprises you? Explain.

Q7 If everything went well, there should be no points in the upper-left part of the graph—a region bounded by a curve. Explain why there are no points in that region.

Now your goal is to figure out the function for that curved boundary. Because calculating area involves squaring, you might reason that the function is quadratic: *Area = Perimeter*^{2}.

10. Select the graph and choose **Plot Function** from the **Graph** menu. In the formula editor, type `Perimeter`^{2} and click **OK.**

11. Unfortunately, the curve is too steep. Double-click the function equation at the bottom of the graph. Change the formula to divide *Perimeter*^{2} by some number, such as *Perimeter*^{2}/3.

Q8 What equation did you try? What happened to the curve—did it get steeper or shallower? Do you think the divisor needs to be bigger or smaller than the number you used?

You can use a slider to try lots of divisors quickly.

12. Drag a new slider from the object shelf. The slider that appears looks like a number line with a pointer on it. It's probably called *V1* and has a value of 5.
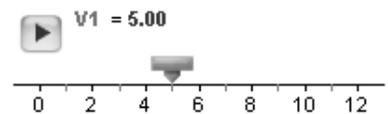
13. Double-click the function equation at the bottom of the graph and change it to use *V1* as the divisor. Now you can control the value of the divisor and change the function by dragging the slider. You will probably need to change the scale of the slider. The easiest way is to drag the numbers on the number line.

Q9 What value of *V1* makes the curve fit the boundary the best?

14. Double-click the collection (the box of gold balls) to show the collection inspector. Then go to the scatter plot of *Area* versus *Perimeter* and click any of the points nearest to the curved boundary. When you do, that case appears in the inspector. Do this for several of the points and notice the *Length* and *Width* of each point.

Q10 What do the cases near the boundary have in common? Why do all those rectangles have that relationship? What type of rectangles are they?

Q11 Use your observation from Q10 and the formulas for area and perimeter to find the boundary curve's function algebraically. Show that it is the same formula you got by using the slider to fit the curve.

EXPLORE MORE

1. There is another empty zone in the lower-right part of the scatter plot of *Area* versus *Perimeter*. Why are there no points in that region? What is the equation of that boundary? Why? Use this and your answer to Q11 to completely describe the relationships between area and perimeter.

2. Look at scatter plots of *Area* versus *Length* and *Perimeter* versus *Length*. Are there maximum and minimum relationships between these attributes? If so, find functions for the boundaries and explain why they exist.