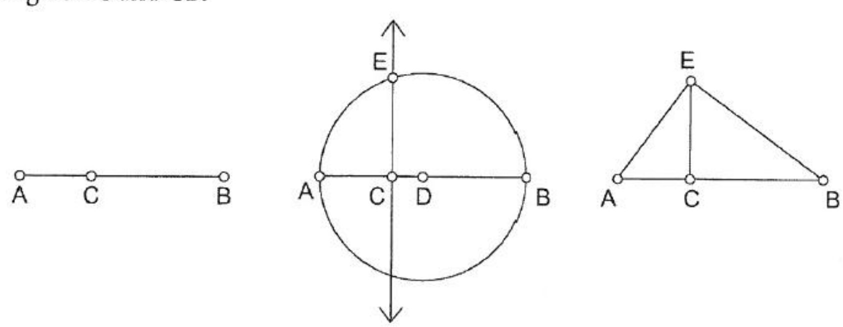
The Geometric Mean

For GSP5

What comes next in the number sequence 2, 6, 18, . . .? If you guessed 54, you noticed that each number in the sequence is just the previous term multiplied by 3. Or you may have noticed that there was a constant ratio between successive terms: $54/18 = 18/6 = 6/2 = 3$. A sequence with a constant ratio is called a geometric sequence, and each term is the *geometric mean* or *mean proportional* between the terms on either side of it. For example, 6 is the geometric mean of 2 and 18 because $2/6 = 6/18$. In this activity you'll discover what a geometric mean is geometrically, and you'll learn how to construct a geometric mean between two lengths.

SKETCH AND INVESTIGATE

1. Construct $\overline{AB}$ and point C on $\overline{AB}$. You will construct the geometric mean of the lengths AC and CB.



Step 1 Steps 2–5 Steps 6–8

2. Construct the midpoint D of $\overline{AB}$.
3. Construct circle DA. Drag point A to make sure the circle is correctly attached.
4. Construct a line through C, perpendicular to $\overline{AB}$.
5. Construct point E where the line and the circle intersect.
6. Construct $\overline{AE}$ and $\overline{EB}$.
7. Hide the circle, the line, point D, and $\overline{AB}$.
8. Construct $\overline{AC}$, $\overline{CE}$, and $\overline{CB}$.

Be sure to release the pointer (or click the second time) with the cursor directly over point A so that the circle will be attached at this point.

Q1 Drag point C. What kind of triangle is $\triangle ABE$? (*Hint:* Recall that it's inscribed in a semicircle.)

Q2 There are three similar triangles in your figure. Write the similarity relationships below and write an explanation of why the triangles are similar.

__________ ~ __________ ~ __________

To measure a ratio, select two segments. Then, in the Measure menu, choose **Ratio**.

Q3 From your similar triangles, figure out which distance is the geometric mean of AC and CB. Measure some ratios to confirm your conjecture. Drag point C to confirm that the distance you found is always the geometric mean. Write a proportion below:

$$\frac{AC}{} = \frac{}{CB}$$

9. Measure CE, AC, and CB.

If you don't see the plotted point, drag the point at (1, 0) to scale the axes.

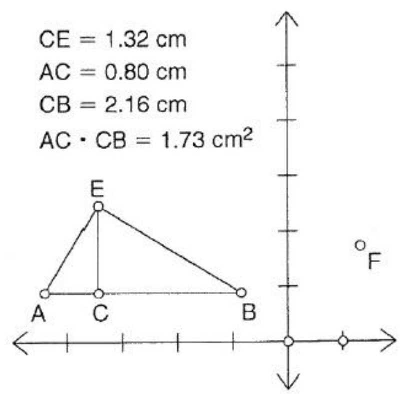
10. Calculate the product $AC \cdot CB$.

11. Select the measurement CE and the product $AC \cdot CB$. Then, in the Graph menu, choose **Plot as (x, y)**.

Select the plotted point and point C. Then, in the Construct menu, choose **Locus**.

12. Construct the locus of this plotted point and point C.

Q4 Describe the graph. Explain how it is related to the proportion you wrote in Q3.



EXPLORE MORE

13. There are two other geometric means in your triangle. Find them and state what they are.

14. Let AE be a, let EB be b, let AC be x, and let CB be $c - x$ (so that AB is c). Use similar triangles to write two proportions. Cross-multiply in each proportion, then add the two resulting equations to combine them into one. Show that $a^2 + b^2 = c^2$ (the Pythagorean theorem).

15. See if you can find a geometric mean in a regular pentagram (five-pointed star).