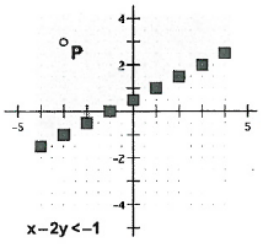
Graphing Inequalities in Two Variables

For GSP5

Imagine yourself as a point in the coordinate plane, free to wander. In this activity you'll travel to different locations, learning how to keep inequalities satisfied.



KEEP ME SATISFIED

1. Open **Graphing Inequalities.gsp.** Move point P around the plane, and observe how the coordinates of the point vary.

Q1 Where can you move the point so that the calculation $2x + 3$ stays greater than 5? Describe these locations.

To edit the calculation, double-click it. Click the y measurement in the sketch to enter it into the calculation.

Q2 Edit the calculation to $2y + 3$. Now where can you move the point so that the calculation of $2y + 3$ stays greater than 5? Describe these locations.

Q3 Press *Show Tracer* and move point P around again, this time anywhere you like. What happens to the tracer when the calculation $2y + 3$ is greater than 5, when it's equal to 5, and when it's less than 5?

Although the inequality $2y + 3 > 5$ has only one variable, we're still interested in all the coordinate points, which have two variables.

A graph of an inequality in two variables is the collection of all the points in the plane that *satisfy* the inequality. In this context, satisfy means "make true." For example, any time point P is in a position where $2y + 3$ is greater than 5, the inequality $2y + 3 > 5$ is true.

Q4 What points satisfy the equation $2y + 3 = 5$? How might knowing this help you describe the points that satisfy the inequality $2y + 3 > 5$?

EXPLORE

To change the calculation, double-click it and then use the Calculator. Enter values of x or y by clicking the x or y measurements in the sketch.

2. Erase the traces, hide the tracer, and change the calculation to $3x - 2y$.

Q5 Given any position of point P, if you move it straight down (without moving it left or right), the value of $3x - 2y$ increases. Explain why. What happens to the calculation if you move P to the right without moving it up or down? Why?

3. Show the tracer and move point P around the plane.

Q6 Describe the locations of point P where $3x - 2y > 5$ is true. Use your answer to Q4 to explain why any point to the right of (or below) the line $3x - 2y = 5$ is a point that satisfies the inequality $3x - 2y > 5$.

4. On page 2, drag point P and notice how it moves between closely spaced points. Show the tracer and drag P again. The tracer changes color, showing for each location of point P whether $x + 2y > -1$ is true.

Q7 Use your answers to Q5 and Q6 to explain why one side of the line $x + 2y = -1$ contains *all* the points where $x + 2y > -1$.

Q8 Without using Sketchpad, use your answers to Q4 through Q6 to predict what points in the plane will satisfy the inequality $3x - y > 2$. Write out your thinking in a sentence, and draw a sketch of the graph on your paper. Then check your answer by editing the calculation and moving point *P*.

Q9 For each inequality below, sketch on your paper your prediction of the graph. Then use Sketchpad to check your result.

a. $-x + 2y > -1$ b. $x + 2y > 1$

c. $-x + 2y > 1$ d. $x - 2y > 1$

Q10 Use page 3 to find points that satisfy the inequality $y < x^2$. How can you determine, without Sketchpad, which points in the plane satisfy $y < x^2$?

Q11 Answer Q10 for the inequality $y > 3x - 5$, again without using Sketchpad.

EXPLORE MORE

To see the entire graph quickly, select both *P* and the tracer square, and choose **Construct | Locus.** Then use **Edit | Properties | Plot** to increase the number of samples so that the locus fills the entire region.

Q12 Use page 4 to find the points in the plane that satisfy the inequalities below. Draw a sketch of each solution. Think about how you could go about predicting the points that satisfy the inequality without Sketchpad available.

a. $x^3 + 4 > 12$ b. $x^3 + 4 < 12$

c. $x^3 + 4 < -12$ d. $|x| - |y| < 3$

e. $|x| - |y| < -3$ f. $|x| - |y| > 3$

Q13 When you're done, compare the solutions to a, b, and c with each other, and the solutions to d, e, and f with each other. What conclusions, if any, can you draw about reversing the inequality sign?

Q14 Use page 5 to find the points that satisfy the inequalities below. How could you predict the points that satisfy the inequality without Sketchpad available?

a. $(x - 2)(x + 1) < 0$ b. $(x - 2)(x + 1)(x - 4) < 0$

Q15 Use page 6 to find points that satisfy the inequality $3x - xy^2 + y^3 < 5$. (Make sure you drag point *P* to a variety of locations.) What points would satisfy the inequality $3x - xy^2 + y^3 > 5$? Why? What points would satisfy the inequality $-(3x - xy^2 + y^3) < -5$? Why? Check your answers with the sketch and explain the results that you see.