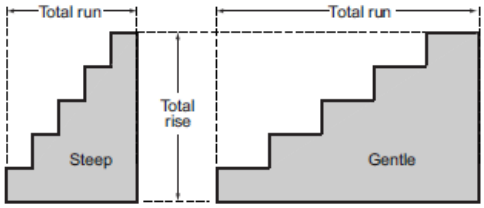
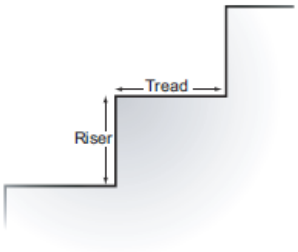
How Slope Is Measured

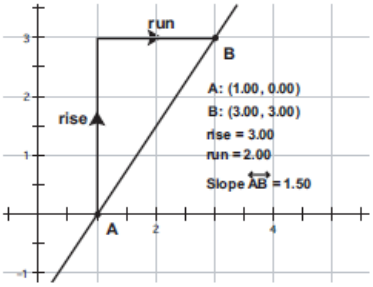


To build a staircase, contractors first need to determine the *total rise* (height) and the *total run* (length) of the staircase. If the total rise is large compared to the total run, the stairs will be steep (and dangerous!). If the total rise is small compared to the total run, the stairs will be easier to climb. So the relationship between the total rise and the total run determines the steepness of the staircase.



Each step also has a rise and run. The vertical part of a step is called the *riser,* and the horizontal part (where you step) is called the *tread.* A step with a large riser and small tread is steep. A step with a smaller riser and longer tread is safer and easier to climb. The steepness of each step depends on the overall steepness of the staircase. As you will see, this way of describing steepness is closely related to how slope is measured.

SKETCH AND INVESTIGATE



1. Open **Slope Measurement.gsp.** Press the *Show Coordinates* button.

Imagine building a staircase on this line, with one step going from point *A* to point *B*.

The rise is like the step riser and the run is like the tread. Press the *Show Staircase* button to see more stairs.

2. Press the *Step* button and observe the rise and the run. Drag *A* and *B,* and watch how these segments and values change.

To keep track of your results using the table in the sketch, press the *Show Table* button. Double-click the table each time you want to add the current measurements to the table.

Q1 For each row of the table, drag *A* and *B* to match the given values, and fill in the rest of the row. (The first row has been filled in for you.)

A: (x_A, y_A)	B: (x_B, y_B)	*rise*	*run*	*Slope* $\overleftrightarrow{AB}$
(2, 1)	(4, 2)	1	2	0.5
(4, 0)	(5, 3)			
(−5, −1)	(−3, 4)			
(−5, 3)	(5, 4)			
(2, −3)	(,)	6	2	

Q2 In Q1, *B* was always above and to the right of *A*, and *rise* and *run* were always positive. What if *B* is below or to the left of *A*? Fill in the table to find out.

A: (x_A, y_A)	*B*: (x_B, y_B)	*rise*	*run*	*Slope* $\overleftrightarrow{AB}$
(2, 1)	(4, 0)			
(1, −1)	(0, 4)			
(−3, 6)	(−5, −1)			
(3, 5)	(,)	−3	−4	

Q3 What happens to *rise, run,* and *Slope* $\overleftrightarrow{AB}$ if you switch *A* and *B*? Try this for some of the table values above. Explain your results.

Q4 What happens to *Slope* $\overleftrightarrow{AB}$ when *B* is above and to the left of *A*? What happens when *B* is below and to the left of *A*? Why do you think this happens?

Q5 Fill in the following table with three other possible locations for point *B*.

If you have a printer and have kept your results in the Sketchpad table, you can use **File | Print** to print the sketch (including the table).

A: (x_A, y_A)	*B*: (x_B, y_B)	*rise*	*run*	*Slope* $\overleftrightarrow{AB}$
(1, 1)	(3, 2)	1	2	0.5
(1, 1)	(,)			0.5
(1, 1)	(,)			0.5
(1, 1)	(,)			0.5

Q6 Describe the locations for *B* that give a slope of 0.5. Explain why you think this happens. Move *A* to a different location. Does your explanation still work?

Q7 Looking back at your tables, you should notice a relationship between *rise, run,* and *Slope* $\overleftrightarrow{AB}$. Write a formula for *Slope* $\overleftrightarrow{AB}$ that uses *rise* and *run.*

Q8 Write a simple formula for *rise* that uses some or all of x_A, y_A, x_B, and y_B.

Q9 Write a simple formula for *run* that uses some or all of x_A, y_A, x_B, and y_B.

Q10 Rewrite your formula for *Slope* $\overleftrightarrow{AB}$ using x_A, y_A, x_B, and y_B.

EXPLORE MORE

Q11 So far, you've thought of *rise* as going up or down from point *A* and *run* as going right or left from there to point *B*. Would the slope be different if you went the other way? Press the *Show B to A* button. You'll see two new segments, *RISE* and *RUN,* going from *B* to *A*. Why is the slope the same whether you go from *A* to *B* along *rise* and *run* or from *B* to *A* along *RISE* and *RUN*?

Q12 In the activity The Slope of a Line, you learned that the slope of any horizontal line is 0 and the slope of any vertical line is undefined. Explain why this makes sense now that you know how slope is measured.