The Isosceles Right Triangle

In this activity you'll discover a relationship among the side lengths of an isosceles right triangle. This relationship will give you a shortcut for finding side lengths quickly. You'll start by constructing a square. Dividing this square in half along a diagonal gives you the isosceles right triangle.
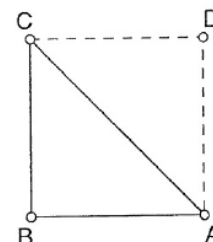
SKETCH AND INVESTIGATE

You can use the tool **4/Square (By Edge)** from the sketch **Polygons.gsp**.

1. Use a custom tool to construct a square *ABCD* by edge endpoints *A* and *B*.

2. Construct diagonal *CA*.

Hide and **Line Style** are in the Display menu.

3. Hide the square's interior, if it has one.

4. Change the line styles of $\overline{CD}$ and $\overline{DA}$ to dashed.

Q1 Explain why △*ABC* is an isosceles right triangle.

Q2 Without measuring, state the measures of the acute angles in △*ABC*.

5. Measure the three side lengths.

Select the hypotenuse and one of the legs. Then, in the Measure menu, choose **Ratio.**

6. Measure the ratio of the hypotenuse length to one of the leg lengths.

7. Drag point *A* or point *B* and observe this ratio.

Q3 What do you notice about this ratio?

In steps 8 and 9, you'll investigate what this ratio represents geometrically.

8. Use the square tool to construct squares on the sides of right triangle *ABC*. Drag to make sure the squares are properly attached.

9. Construct one diagonal in each of the smaller squares, as shown at right.

Q4 The diagonals you drew in the smaller squares may help you see a relationship between the smaller squares and the square on the hypotenuse. If each of the smaller squares has area x^2, what is the area of the large square? Confirm your conjecture by measuring the areas. Drag the triangle to confirm that this relationship always holds true.
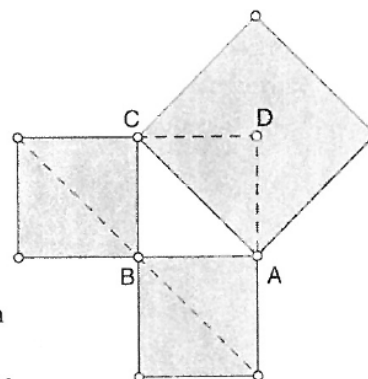
Q5 In an isosceles right triangle, if the legs have length x, what is the length of the hypotenuse?

Q6 Use the Pythagorean theorem to confirm your answer to Q5.

EXPLORE MORE

10. The society of the Pythagoreans discovered that the square root of 2 is *irrational*. Do some research and report on this discovery.