

SKETCH AND INVESTIGATE

- Q1** Triangle ABC is right and isosceles because two of its sides are sides of a square. Thus, they're congruent and they form a right angle.
- Q2** Each acute angle in $\triangle ABC$ measures 45° .
- Q3** The ratio is constant at 1.414. (Decimals will vary depending on the precision students have set in Preferences.)
- Q4** If each of the smaller squares has an area of x^2 , the area of the large square is $2x^2$. (Each small square can be divided into two isosceles right triangles, four of which fit into the large square.)
- Q5** If the legs of the isosceles triangle have length x , the length of the hypotenuse is $x\sqrt{2}$. This is because the area of the square on the hypotenuse is $2x^2$, so the length of a side of the square must be, $\sqrt{2x^2}$ or $x\sqrt{2}$.
- Q6** According to Q5, if each leg has length x , the hypotenuse has length $x\sqrt{2}$. Substituting these values in the Pythagorean theorem gives $x^2 + x^2 = (x\sqrt{2})^2 = 2x^2$.

EXPLORE MORE

- 10. The discovery of the irrationality of $\sqrt{2}$ originated with the isosceles right triangle with sides of length 1. The Pythagoreans were startled to find a number that could not be represented as the ratio of two integers. The standard proof of the irrationality of $\sqrt{2}$ is by contradiction: assume the number is rational and show that this leads to impossible conclusions. The tenth book of Euclid's *Elements* contains a discussion similar to this proof. And a reference in one of Aristotle's works makes it clear that the proof was known much earlier than Euclid.

The Isosceles Right Triangle



In this activity you'll discover a relationship among the side lengths of an isosceles right triangle. This relationship will give you a shortcut for finding side lengths quickly. You'll start by constructing a square. Dividing this square in half along a diagonal gives you the isosceles right triangle.

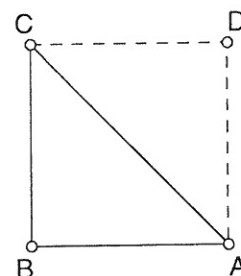
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You can use the tool **4/Square (By Edge)** from the sketch **Polygons.gsp**.

Hide and Line Style are in the Display menu.

Select the hypotenuse and one of the legs. Then, in the Measure menu, choose **Ratio**.

1. Use a custom tool to construct a square $ABCD$ by edge endpoints A and B .
2. Construct diagonal CA .
3. Hide the square's interior, if it has one.
4. Change the line styles of \overline{CD} and \overline{DA} to dashed.



Q1 Explain why $\triangle ABC$ is an isosceles right triangle.

Q2 Without measuring, state the measures of the acute angles in $\triangle ABC$.

5. Measure the three side lengths.
6. Measure the ratio of the hypotenuse length to one of the leg lengths.
7. Drag point A or point B and observe this ratio.

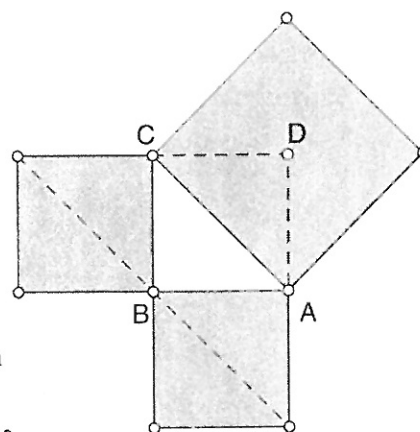
Q3 What do you notice about this ratio?

In steps 8 and 9, you'll investigate what this ratio represents geometrically.

8. Use the square tool to construct squares on the sides of right triangle ABC . Drag to make sure the squares are properly attached.
9. Construct one diagonal in each of the smaller squares, as shown at right.

Q4 The diagonals you drew in the smaller squares may help you see a relationship between the smaller squares and the square on the hypotenuse. If each of the smaller squares has area x^2 , what is the area of the large square?

Confirm your conjecture by measuring the areas. Drag the triangle to confirm that this relationship always holds true.



- Q5** In an isosceles right triangle, if the legs have length x , what is the length of the hypotenuse?
- Q6** Use the Pythagorean theorem to confirm your answer to Q5.

EXPLORE MORE

10. The society of the Pythagoreans discovered that the square root of 2 is *irrational*. Do some research and report on this discovery.