

# Chapter 1: Exploring Triangles

**Summary:** In this lesson, students will explore properties of triangles while becoming familiar with a variety of different features of dynamic geometry environments and pedagogical issues that arise when students are learning geometry.

**Objectives:** At the end of this chapter students will be able to:

## *Mathematical*

- Provide a description of geometry and make distinctions between geometry and measurement.
- State the triangle inequality and provide applications of its use.
- Determine whether two different statements of a theorem are equivalent.
- Classify different triangle centers and describe how they can be used in solving real-world problems.

## *Technological*

- Describe and use common features of dynamic geometry environments. These features include: parent/child relationships, dragging, measuring, constructing/drawing, and calculating.
- Know that dragging different points of a geometrical figure can result in different behaviors.
- Create different constructions of a single geometrical object using a DGE.
- Use technology to solve problems involving triangles and triangle centers.

## *Pedagogical*

- State common misconceptions students have related to interpreting representations of geometrical objects such as angles.
- Be familiar with different ways a single geometrical object can be classified and the implications this has for teaching.
- Explain how different tools can shape the ways in which students think about particular geometrical objects.
- Critique a mathematical task and sketch provided to students for teaching the Triangle Inequality Theorem.
- Analyze how students interact with a task and a technology tool to learn a new mathematical idea.
- Describe the difference between evidence and proof and how teacher's can support students' use of evidence in explanations students provide.
- Create new tasks from existing tasks.

**Prerequisites:** Students should be familiar with triangles, circles, and properties of parallel and perpendicular lines.

**Vocabulary:** Parent/Child relationships, Dragging, Constructing/Drawing, Drag Test, Triangle Inequality Theorem, Conjecture, Rigid Figures, Incenter, Circumcenter, Centroid

**Technology and Video Files:** Triangle\_Inequality.gsp; RightTriangle.mov;  
TriangleInequality.mov; Map\_Triangle.gsp

**Website links to technology files:** INSERT WEBSITES

**Materials Needed:** a computer with internet access, The Geometer's Sketchpad (optional), a  
Dynamic Geometry Environment

## Chapter 1: Exploring triangles

### Section 1: What is Geometry?

Most people have had experience learning geometry in elementary, middle, and high school. Points, lines, planes, triangles, squares, and rectangles, are just some of the objects that one might study. But what exactly is geometry? The Greek derivation of the word, geo-metry, literally means “earth measure.” But rarely do we associate the study of geometry with measures of the earth. What do you think of when you hear the word “geometry”?

#### ENGAGING WITH CONTENT

- Q1. When you hear the word “geometry” what do you think of? Create a list of different terms and arrange them into a concept map.
- Q2. Find other descriptions of geometry on the internet. Describe some similarities and differences in the descriptions.
- Q3. How is geometry similar to and different from measurement?

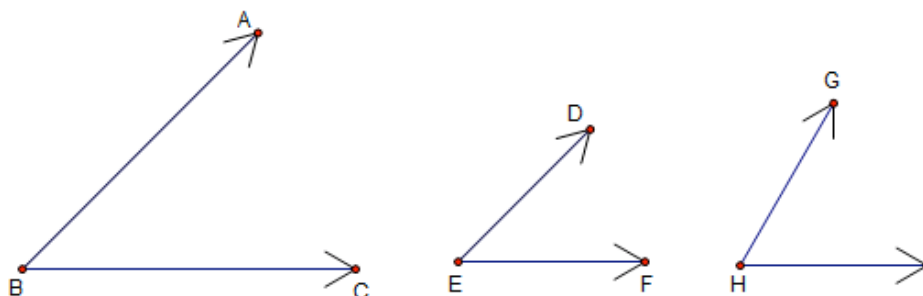
#### CONSIDERATIONS FOR TEACHING

- Q4. What are some important ideas for students to study when they are learning geometry in middle school and high school? Explain why you believe these ideas are important.

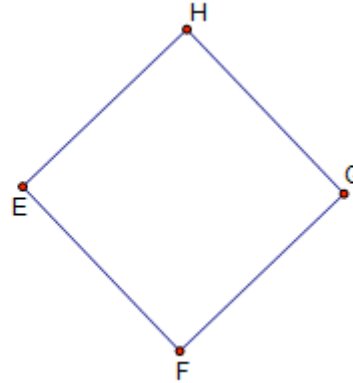
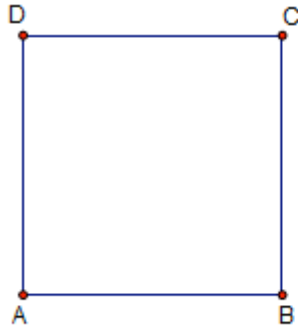
## Section 2: Geometry and Representations

When we think about the geometry learned in school, it is generally connected with the study of abstract ideas such as points, that have no dimension, or lines of one dimension that go on forever. These objects do not exist in the world around us, but can only be imagined in the mind. But people use these objects all the time. For example, an architect might use points to develop a plan for a building or a computer-aided designer might use line segments in drawings of complicated mechanical parts to be machined. In each case, a representation of a point or a line segment must be created. These representations differ from the geometrical objects they represent. Any representation highlights some aspects of an object and obscures others. For example, a dot highlights that a point is a location in space, but obscures the idea that a point does not have any dimension. In the study of geometry, it is important to consider differences between abstract geometrical objects and physical representations that are used.

Students often have difficulty reasoning from representations of different geometrical objects. For example, when students are first learning about angles, they often focus on the represented lengths of the rays that form the angles rather than the degree measure of the angle. For example, when presented with representations of three different angles, shown below, students often state that  $\angle ABC$  is the largest because its represented rays are longer than the represented rays of the other two angles.



Students also tend to associate figures with particular representations. For example, a student may believe quadrilateral  $ABCD$  is a square, because the way it appears is familiar and looks like a square most students are accustomed to seeing. However, if the square is rotated 45 degrees (quadrilateral  $EFGH$ ) they may describe it as a diamond rather than a square even though it still has four congruent sides and four right angles.

**CONSIDERATIONS FOR TEACHING**

Q5. Describe some strategies teachers can use with students to assist them in attending to relevant, rather than irrelevant features, of geometrical representations (for example, the congruent angles and sides of a square rather than its appearance or position).

Q6. In the videoclip, “RightTriangle.mov” you will see two students who are considering the following question, “Can an isosceles triangle have exactly two 45 degree angles?” Play the movie and then respond to the following questions.

- A) Describe what the student on the left may have noticed on the computer screen.
- B) Why do you think the student on the left is having difficulty seeing the right triangle?
- C) How did the student on the right respond to her partner’s question? Do you think this was an appropriate response or should she have responded differently?
- D) If you were the teacher and you just observed this interaction, would you follow-up with a question? If yes, what would you ask? If no, why not?

Geometry is a visual subject. It is difficult to imagine thinking geometrically without drawing a picture or using some sort of visual aid. But like representations, pictures can sometimes be difficult for students to interpret. For example, consider the following picture.



What is the classification of the object above?

This question seems straight forward. However, there are at least six different correct answers one could provide: A simple closed curve, convex polygon, quadrilateral, parallelogram, trapezoid (depending how it is defined), and rectangle. The way we identify an object is usually connected to the context. We might consider what topics or ideas were included in the chapter of the book or questions such as these that were posed during a lesson prior to providing a response. So even when we create a picture of a geometrical object, the object the picture is intended to represent may not be obvious to students who are just learning geometry.

**Tech Tip:****ENGAGING WITH CONTENT**

Q7. Create an example of a geometrical object that can only be classified in one way. Is it possible? What about an object that can only be classified in two ways?

**Tech Tip:****ENGAGING WITH CONTENT**

Q8. Although most geometric shapes have multiple classifications, students are generally expected to provide one correct classification. What are the advantages and/or disadvantages to an approach to classifying shapes that focuses on a single classification versus multiple classifications?

### Section 3: Dynamic Geometry Environments

Dynamic geometry environments (DGEs) are particular technology tools that have been used in the learning and teaching of geometry to assist students in moving beyond specifics of a single drawing to generalizations across figures. There are over forty different DGEs. Although the programs are all different, they share common features. Some of these features include: a set of primitive objects (e.g., points, lines, rays, circles), the ability to construct parallel and perpendicular lines, tools for performing transformations and calculations, dragging, measuring, animation, and an ability to hide objects, and create procedures. This section will focus on three key features: parent-child relationships, dragging, and constructing/drawing.

#### *Parent-Child relationships*


There are usually at least two different ways to create a point on an object. You can select the point tool and move the cursor to the segment and then click to create a point on the segment or you can use commands from the menu. The first method relies on mouse precision and the point may not be on the segment, but only close. The second method is generally more accurate.

1. Open a new sketch in your dynamic geometry program. Create and label  $\overline{AB}$ .




2. You may use the following directions:



- i) Click on the segment tool . Then, in the workspace, click to mark the first endpoint and drag until a segment appears. Click again to mark the second endpoint.

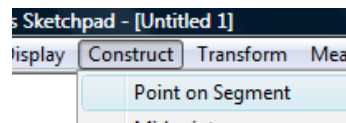
**A**

- ii) Move your cursor to the toolbar and click on the text tool . Then, move your cursor until the hand is pointing at the endpoint and becomes dark in color.




By default, GSP will label the points  $A$  and  $B$  unless these labels have been previously used. If you wish to change the label, simply double click on the label, rename it in the window that appears and click OK.

- Construct point  $C$  as a point on  $\overline{AB}$  by highlighting the segment and choosing “Point on Segment” from the Construct menu. Label this point  $C$  using the directions above (step 1, ii).



Drag points  $A$ ,  $B$ , and  $C$  and notice what changes and what stays the same.


Because point  $C$  was created as a point on  $\overline{AB}$  it can only move along that segment. However, points  $A$  and  $B$  were arbitrarily placed in the plane and can be moved to any location. Point  $C$  may be referred to as a “child” of or stated another way, segment  $AB$  is the “parent” of point  $C$ .

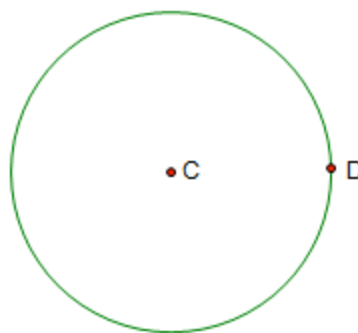
- The child depends on the parent. Click on the selector tool , click on point  $A$ , point  $B$  or  $\overline{AB}$  then press the delete key on your keyboard. Notice that point  $C$  is also deleted because it depends on  $\overline{AB}$  which depends on points  $A$  and  $B$ .


### Dynamic Behavior

Another important feature of DGEs is the way in which different objects behave when dragged. Let's consider the example of a circle.

You can generally undo a command by selecting “Undo” from the Edit menu.

- Begin with a new sketch. Choose the circle tool . Then, in the workspace, click to mark the center of the circle and drag until a circle appears. Click again to mark a radius point. Use the text tool to label the circle with center point  $C$  and radius point  $D$ .



- Click on the selector tool , then try three different ways to drag this circle, with each method resulting in different behaviors. For example, you can click on point  $C$  and drag it. Notice how the circle changes. Or you can drag point  $D$ . Notice the center remains fixed but the size of the circle



changes. You can also drag the circumference of the circle. The entire circle changes location but its size remains the same.

### ENGAGING WITH CONTENT

Q9. How is creating a circle with the circle tool similar to and/or different from creating a circle with a compass? [From this point on in the text we will use the term circle to refer to a representation of a circle]

Q10. How are parent/child relationships similar to independent and dependent variables that are often introduced to students in algebra?


### CONSIDERATIONS FOR TEACHING


Q11. Why is it important for students to know that dragging different points of a figure may affect how it changes?

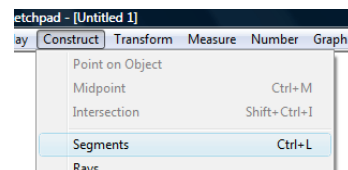
Q12. It has been stated that the use of tools can shape the way students think. As an example, describe how drawing a circle freehand with a pencil requires different thinking than drawing with a compass or drawing with a DGE. What features of a circle are highlighted by each of these tools and how might a student think about a circle differently as a result?

Q13. Use the ray tool to create an angle. Notice how your DGE represents it. How is this similar to and/or different from the way angles are generally represented using paper. How could this representation influence how students think about angles?

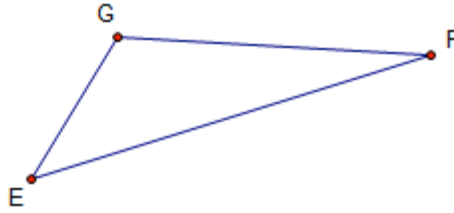
Although noticing how a single object can change is interesting, the drag feature can also be used to create special cases and assist one in looking for relationships.

1. Begin with a new sketch. Select the point tool  and click in the

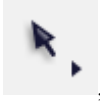
workspace to create three points. Next, choose the selector tool , and click on the three points. Then choose Segments from the Construct menu to create a triangle. The Ctrl+L shortcut can be used for

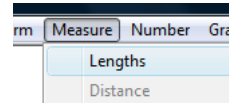


the same construction. Now, label the three vertices  $E$ ,  $F$ , and  $G$  using the text tool.



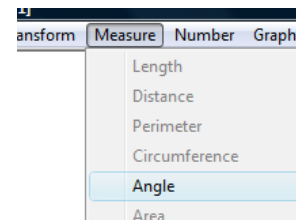
2. Use your DGE to measure the lengths of the sides of the triangle. Choose the

selector tool , and click on each of the three segments of the triangle. Then, select Lengths from the Measure menu. Lengths for three segments will appear in the workspace.



3. Notice that as you drag a vertex of the triangle the side lengths change. Drag points until you create a triangle that is approximately scalene, isosceles, and then equilateral.

4. It is also useful to measure angles of triangles. To measure an angle, select the three vertices, then choose Angle from the Measure menu. The angle measure will appear. Remember, the order in which you select the vertices is important in identifying which angle you are measuring. Measure the three angles. Drag points until you create a triangle that is approximately acute, obtuse, or right.



### \*\*\*TECH TIP\*\*\*

Sometimes you may want to add captions to your sketch. This is easy to do. Simply click on the text tool and draw a text box in the sketch. Then, type the caption you want to appear. Captions can also contain Hot Text™ links to object labels and values of your sketch. The links are created by simply clicking on the object or value you want to use. These Hot Text™ values change dynamically when the sketch is changed.

Use the sketch you created by the steps above and try the following example:

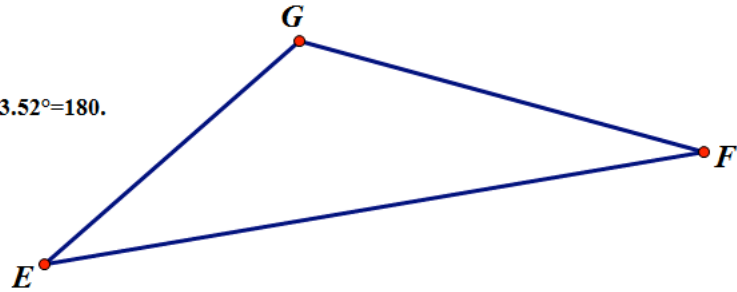
- Click on the text tool and draw a text box in the sketch.
- Type *The sum of*. Then, rather than typing the angle names or measures, click on the angle measures in the sketch. Finish the sentence with  $= 180$ .

$$m\angle GEF = 31.51^\circ$$

$$m\angle EFG = 24.97^\circ$$

$$m\angle FGE = 123.52^\circ$$

$$\text{The sum of } 31.51^\circ + 24.97^\circ + 123.52^\circ = 180.$$



- c) Finally, drag the points of the triangle. Notice the values in the text box changing dynamically.

### ENGAGING WITH CONTENT

Q14. Which geometrical properties of triangles could you have students explore using this sketch?

### CONSIDERATIONS FOR TEACHING

Q15. Describe how the process of finding the measure of an angle with the DGE can assist students in categorizing angles?

Q16. In section 2, difficulties students have in recognizing squares and other figures that are not in “standard” position were described. How can a teacher use a DGE to assist students in recognizing and classifying shapes when they are located in different locations in the plane?

Q17. Students are often asked to classify different triangles and consider relationships among these classifications (e.g., can a right triangle also be isosceles?) Create a list of questions that you could pose to students that they could investigate using this sketch.

Dragging the points of a general triangle to make a right triangle is nearly impossible as the software will only recognize the measures of the angles as approximations. If we want to create a triangle that is a right triangle and will always be a right triangle, then it is best to construct a triangle to be right rather than draw a triangle and drag a vertex. A **construction** involves the use of properties of the figure in its creation. Whenever a portion of the diagram is dragged, the construction will maintain the properties that were used to construct it. A **drawing** involves the use of “free-hand” tools and the properties will not be

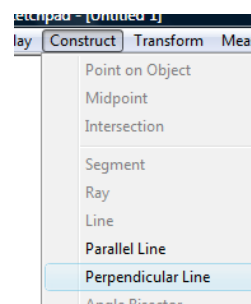
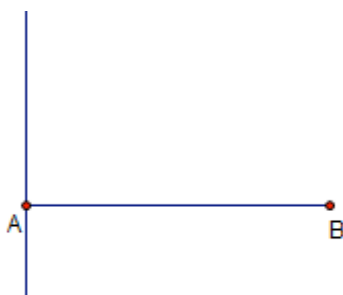
maintained under the drag mode. The triangle that was created and dragged to become a right triangle was a drawing. Let's see how to construct a right triangle.

1. Draw  $\overline{AB}$ .

2. A perpendicular line is needed to create a right angle. To construct a line perpendicular to  $\overline{AB}$  that passes through point  $A$ , use the selector tool

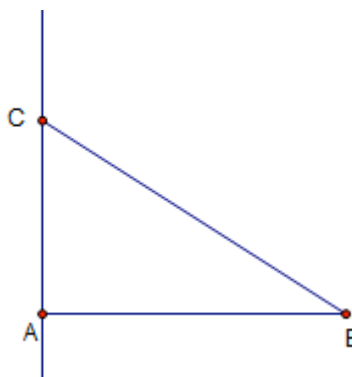


, and click on  $\overline{AB}$  and point  $A$ . Then select Perpendicular Line from the Construct menu.



3. Construct a point on the perpendicular line and label that point  $C$ . If you drag point  $C$ , notice that it will always be on the perpendicular line.

4. Points  $A$ ,  $B$  and  $C$  will be the vertices of the triangle. Click on points  $A$ ,  $B$ , and  $C$ . Then use the Ctrl+L shortcut (or the segment tool) to create  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$ .

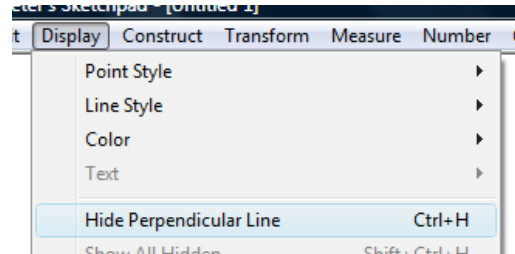


5. You may clean up the sketch by hiding the perpendicular line. To do this,

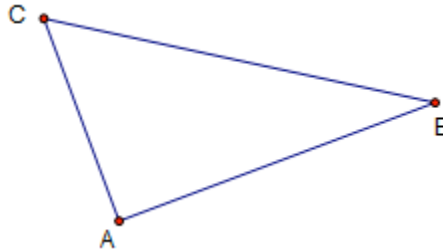


use the selector tool, and click on  $\overline{AC}$ . Then, select Hide

Perpendicular Line from the Display menu. Notice, there is also a Ctrl+H shortcut for the “hide” feature. Remember, if it is deleted, point  $C$  will also be deleted since  $C$  is a child of the perpendicular line. This right triangle is considered a



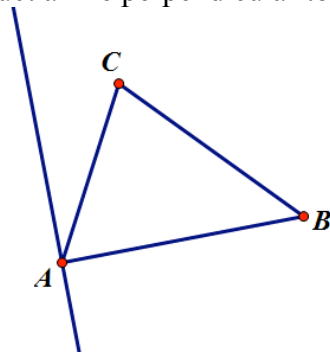
construction, because when any point of the object is dragged, the triangle continues to be a right triangle. Selecting a point of the figure and dragging it is also a good way to determine whether a figure has been constructed or drawn. This is often referred to as a “**drag test**.” It is a useful test for both students and teachers for checking a construction.




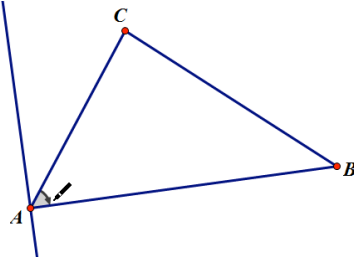
**\*\*\*TECH TIP\*\*\***

As with many GSP constructions, there is usually more than one way to do things. Consider this alternate method for constructing a right triangle:

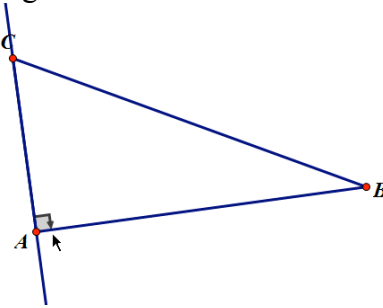
- Construct triangle  $ABC$ .
- Construct a line perpendicular to  $\overline{AB}$  through point  $A$ .



- c) (This step is optional.) Click on the marker tool , then click on  $\overline{AC}$  and drag the mouse to  $\overline{AB}$  to mark  $\angle CAB$ . Notice the angle marking in the sketch below.



- d) With the perpendicular line highlighted, click on point  $C$ . Then, choose Merge Point to Perpendicular Line from the Edit Menu. Notice  $m\angle CAB = 90^\circ$  and that the angle marking (from part c) becomes a right angle marking.



### ENGAGING WITH CONTENT

Q18. What geometrical properties of right triangles could you have students explore using this sketch?

Q19. Make a list of different properties of an isosceles triangle? Use your DGE to construct, not draw, an isosceles triangle. Then, use your DGE to construct the triangle using a different method.

### CONSIDERATIONS FOR TEACHING

Q20. Why is it important for students to understand the difference between drawing and constructing?

Q21. How might a student think differently when constructing rather than drawing a figure?

Q22. Notice that different construction methods result in figures that look the same, but behave differently when a point of the figure is dragged. Considering the

“behavior” of figures is a new feature introduced by dynamic environments. Discuss how a focus on behaviors of geometric figures when they are dragged can assist students in thinking about properties.

## Section 4: Exploring Lengths of Sides of Triangles

When studying triangles it seems natural to begin by asking, “what conditions are needed to create a triangle?” Most students will respond, “three sides and/or three vertices.” However, it is interesting to think about whether any three side lengths or any three points can be used to create the sides and vertices of a triangle.

### ENGAGING WITH CONTENT

Q23. If you are given any three points in the plane, will they form vertices of a triangle? Explain your answer and feel free to use your DGE.

Q24. If you are given any three segments, can they be arranged to form a triangle? Explain your answer and feel free to use your DGE to explore.

The relationship among sides in a triangle that you investigated in M-Q24 is often difficult for students to understand. The Triangle Inequality Theorem provides information about the constraints on the side lengths that must be met in order to form a triangle. A statement of the theorem follows:

**Triangle Inequality Theorem:** In a triangle, the sum of the lengths of any two sides is greater than the length of the third side.

Some students recall this theorem as: The sum of the lengths of the two shortest sides must be greater than the length of the longest side.

### ENGAGING WITH CONTENT

Q25. Is the second statement of the theorem equivalent to the first? Explain.

### CONSIDERATIONS FOR TEACHING

Q26. Which statement would you want to provide to students? Explain.

Q27. Suppose you are planning to introduce students to the triangle inequality theorem using a DGE. Design a sketch that students could use to discover this theorem and create a list of guiding questions you could pose to students as they interact with the sketch.



Q28. Open up the sketch Triangle\_Inequality.gsp. In what ways could this sketch be helpful to eighth grade students who are learning the triangle inequality theorem. How might the sketch confuse students? What changes, if any, would you suggest be made to the sketch.

A teacher, Mr. Smith, taught a class of eighth graders the triangle inequality theorem using the sketch that you explored and critiqued in Q-28. He also provided students with a handout that is shown below.

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### Triangle Inequality

(Adapted from Geometry Activities for Middle School Students with the Geometer's Sketchpad, 1998)

Open the sketch Triangle\_Inequality.gsp

Try to make a triangle using the lengths of sides  $a$ ,  $b$ , and  $c$  in the table below. To adjust the length of  $a$ ,  $b$ , or  $c$ , drag the right most endpoint of the parallel segments labeled “side  $a$ ”, “side  $b$ ”, or “side  $c$ ”. Then, swing the endpoints of the figure to see whether you can make a triangle. The endpoints must meet to form the vertices of the triangle. If a triangle is formed draw a picture of it in the space provided. If a triangle cannot be formed, write *impossible*.

#	Length of Side $a$	Length of Side $b$	Length of Side $c$	Triangle?
1	2.0 cm	3.0 cm	4.0 cm	
2	6.0 cm	1.0 cm	4.0 cm	
3	3.5 cm	2.0 cm	6.0 cm	
4	3.0 cm	4.0 cm	4.0 cm	
5	5.0 cm	5.0 cm	6.0 cm	
6	2.0 cm	7.0 cm	4.0 cm	

Why was it impossible to construct a triangle with some of the given lengths?

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Write a conjecture about the relationship among the lengths of the three sides of a triangle.

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### ENGAGING WITH CONTENT

Q29. Consider the six sets of side lengths that were provided to students on the handout. Use the sketch to determine which ones will work and which ones will not.

### CONSIDERATIONS FOR TEACHING

Q30. Develop a rationale for why the teacher selected the particular examples that he did. Are there any that you would change? Explain.

Q31. Are there any of the sets of side lengths that you think students will have difficulty with? What explanations and conjectures do you anticipate students will create?

Open the videoclip, “TriangleInequality.mov” and watch the first section “Beginning of the Lesson: Teacher gives Directions” (0:00 – 2:43). Stop the movie and then answer questions 33-36 below.

### CONSIDERATIONS FOR TEACHING

Q32. The teacher made a decision to open and display the sketch to the class, explain some of its components and work through the first example on the worksheet. Do you agree with this decision? In general, when do you think a teacher would want to work through an example with the class and when do you think he/she would not want to do so?

Q33. During this episode there is a 30 second pause (1:06-1:36). What do you think the teacher and students are doing during this pause? Do you believe it was important for the teacher to pause at this time?

Q34. When the teacher dragged the endpoints of the sides of the triangle and asked “Do you think they’re going to meet?” many students said, “no.” Was this surprising? Why do you think they responded with this answer?

Q35. What else did you notice about what the teacher said or did during the beginning of this lesson that you found interesting?

Start the movie again and watch the last remainder of the movie (beginning at 2:44). You will see two eighth grade mathematics students working together on the worksheet and at the same time you can view what they are doing on their computer. James is sitting on the left and Kristy is sitting on the right.

### CONSIDERATIONS FOR TEACHING

Q36. The students are working in pairs and sharing a laptop. What are the advantages and disadvantages of providing these students with just one rather than two laptop computers?


Q37. What roles or jobs related to solving the problem has each student assumed? Are they working effectively as a pair?

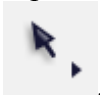
Q38. In response to the question, “Why was it impossible to construct a triangle with some of the given lengths?” (5:45) James stated, “Because one side was too long or too short. Is his answer correct? If you were the teacher, how would you respond?”

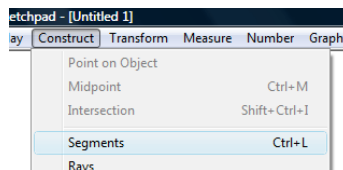
Q39. The conjecture (6:29) they develop about a relationship among the lengths of the sides of the triangle is “if there’s a long length, that’s your bottom, and so the two lengths on the sides, they got to be very close to each other.” What is your interpretation of what James’ said? What question could you pose to James to assist him in making connections between this statement and the Triangle Inequality Theorem?

Q40. How did the students use the sketch? Was it helpful to them?

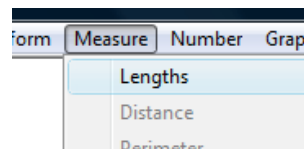
**Problem Posing.** You have had an opportunity to explore the triangle inequality theorem and also to observe what students consider when they are investigating this theorem. A natural next step would be to pose a new problem. Creating new problems can often be just as challenging as solving given problems, but there are some questions you can consider when thinking about how to modify or extend a problem. For example you can ask the question, “**What if not?**” This problem considers a relationship among lengths of sides using an inequality. What happens when the sum of the lengths of two sides is not greater than the length of the third side (equal to or less than)? Another approach is to **consider the attributes** of the given problem. In this problem we were given a triangle and the lengths of two sides. What if we considered a quadrilateral or an n-sided polygon and the sum of the lengths of n-1 sides? Let’s consider the case of quadrilateral.

1. Begin with a new sketch. Select the point tool , and click in the white space to create four points. Next,

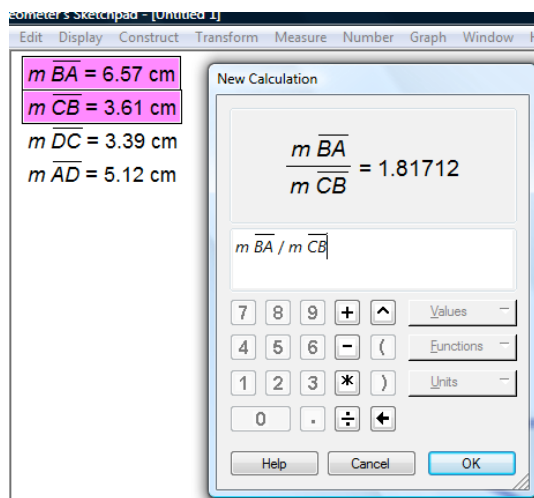
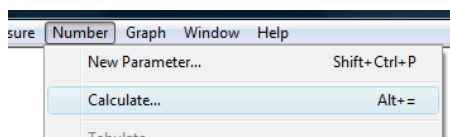
choose the selector tool , and click on the four points. Then, choose Segments from the Construct menu to create a quadrilateral. Remember, the Ctrl+L shortcut can be used for the same construction. Now, label the three vertices  $A$ ,  $B$ ,  $C$ , and  $D$  using the text tool.



2. Click on each of the four segments to highlight them. Then, select Lengths from the Measure menu.



3. Explore relationships among the lengths. What do you notice? You may wish to use the Calculate tool under the Number menu. For example, suppose you wish to know the ratio of the lengths of  $\overline{BA}$  and  $\overline{CB}$  (see screenshot below).



**ENGAGING WITH CONTENT**

Q41. Is there a relationship among the lengths of the sides of a quadrilateral? If yes, state the relationship. If no, explain.

Q42. How are you convinced that the answer is yes or no.

**The role of evidence in DGEs.** You may recall from high school or college mathematics courses that one example can be used to disprove a statement (a counterexample), but usually one or more examples cannot be used to prove that a statement is always true. In DGEs, it is easy to generate multiple examples by dragging and this is often convincing enough for students to accept that a statement or conjecture is always true. However, teachers need to make sure that students understand differences between examples and proofs.

**ENGAGING WITH CONTENT**

Q43. Consider your investigation of the lengths of the sides of a quadrilateral. If you stated that there was a relationship among the lengths of sides, how did you convince yourself this was true? How would you prove it is always true? If you stated there was not a relationship, what did you do to arrive at this conclusion?

**CONSIDERATIONS FOR TEACHING**

Q44. How can you assist students in understanding the difference between evidence and proof when they are using DGEs? What tasks or questions could you pose?

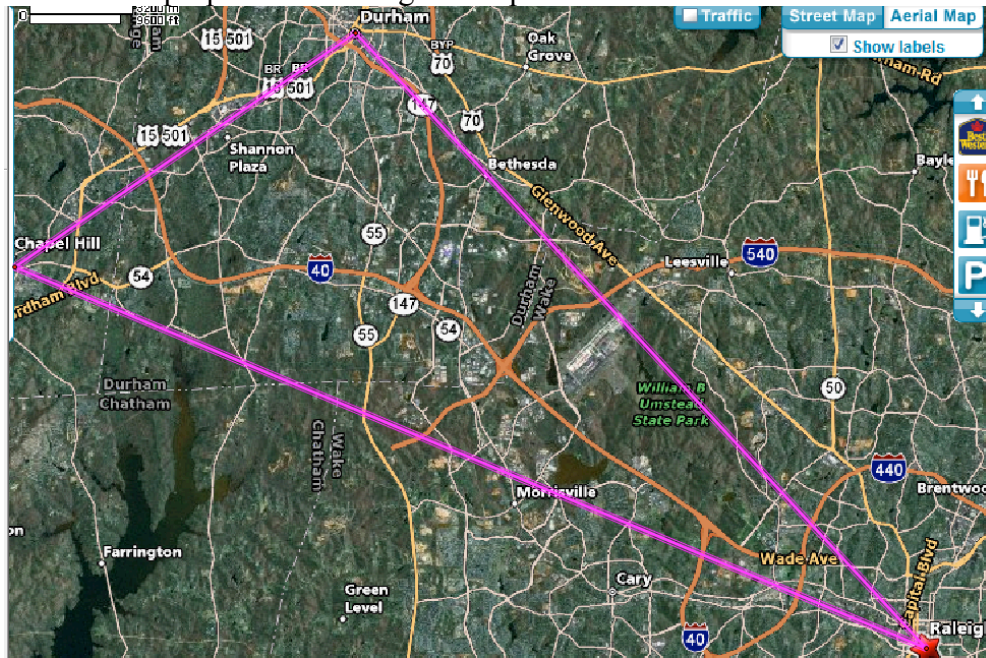
Q45. Suppose a student used a DGE to create three non-collinear points,  $A$ ,  $B$ , and  $C$  and connected the points with segments to create triangle  $ABC$ . Next, the student constructed the bisector of  $\angle ABC$  and stated, "For any triangle the angle bisector passes through the midpoint of the opposite side."

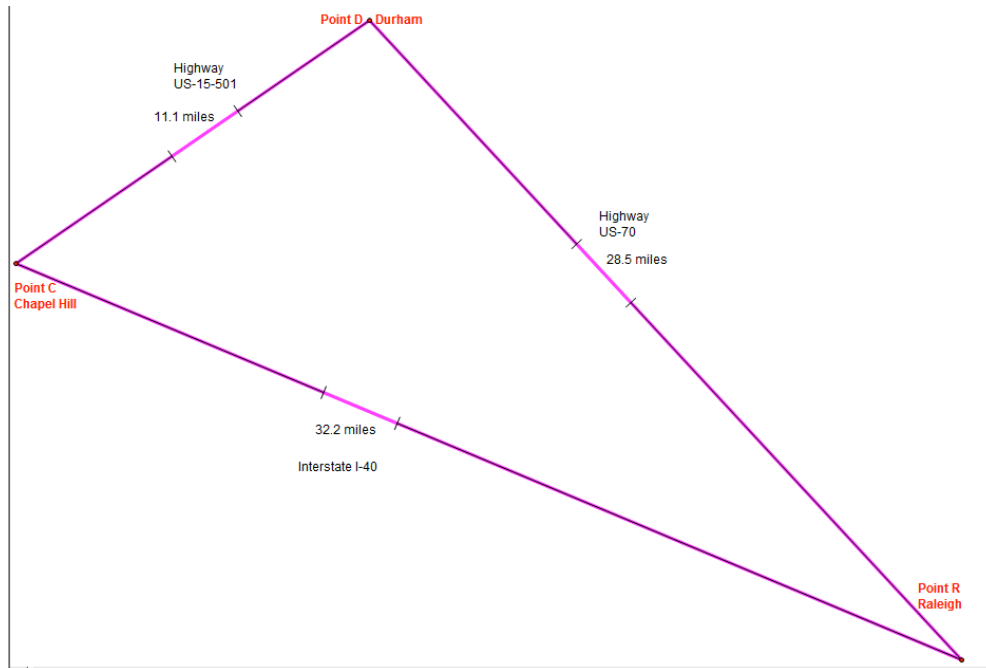
Is this statement correct? If it is not correct, what do you think the student did with the DGE that led to this statement? How would you respond to this student?

## Section 5: Using Triangles to Solve Problems

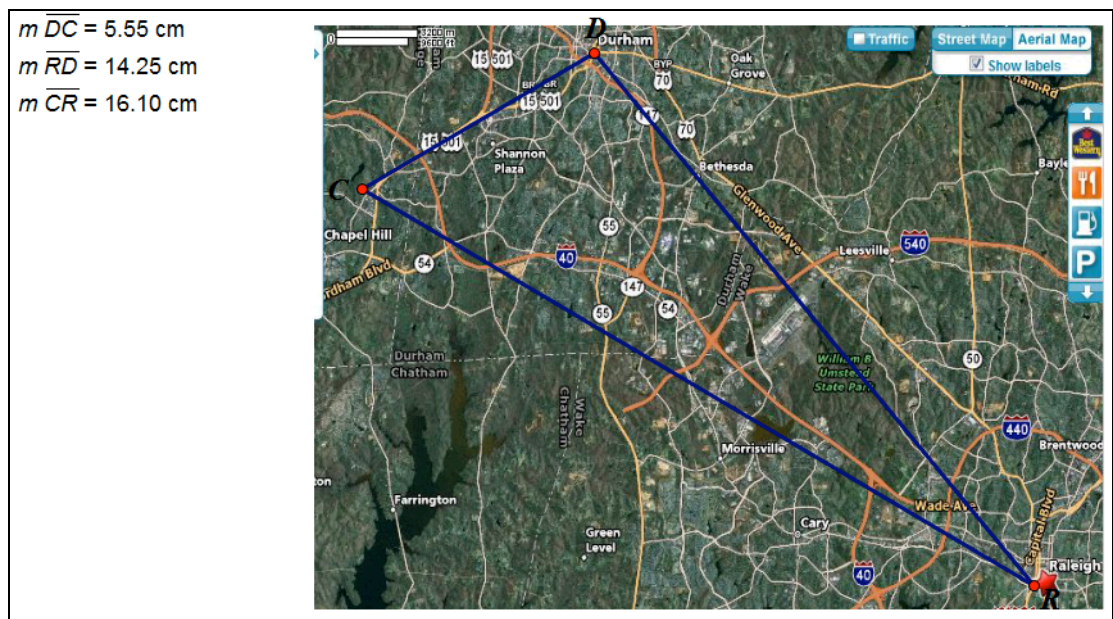
Triangles are also important to study because they can be used to solve more complex problems. Consider the following problem:

**Building a New Stadium.** In the state of North Carolina, a specific region in the Piedmont area is known as the “Research Triangle” or just “Triangle” because its area is contained between three cities: Raleigh (approximate population: 380,173), Durham (approximate population: 479,624), and Chapel Hill (approximate population: 54,492). Suppose the state is planning on bringing in a professional baseball team and needs to build a new baseball stadium for it. An aerial map of the area is shown below with a second more simplified map of the area that can be used for the purpose of working on the problem.





Imagine you are on the board that will decide on where the stadium will be located and the planner has presented two options. You can use the triangle\_map.gsp file (see screenshot below) to help you consider the following two options.



Option 1: Place the stadium the same distance from each of the three cities. If this option is selected the state will need to pay to build three new highways from each city to the stadium.

Option 2: Place the stadium equidistant from the three existing highways. The existing highways would need to be resurfaced and new highways built from each existing highway to the stadium.

Assume the cost of building new highways is \$125,000 per mile and the cost of resurfacing an existing highway is \$50,000 per mile.

### ENGAGING WITH CONTENT

Q46. For each option, describe how you used your DGE to determine the location of the stadium. Include a screenshot that shows the three cities and the location for the stadium. Also, for each option compute the cost for road construction/resurfacing.

Q47. Which option would you choose? Explain.

Q48. The planner has also suggested a third option that could take into consideration the populations of the three cities. Describe a possible option that uses population information to find a site for the stadium.

Q49. Create a different, fourth option? Describe and explain how it compares to the other three options.

### CONSIDERATIONS FOR TEACHING

Q50. How is this task related to common topics that are studied in a high school geometry course? Explain.

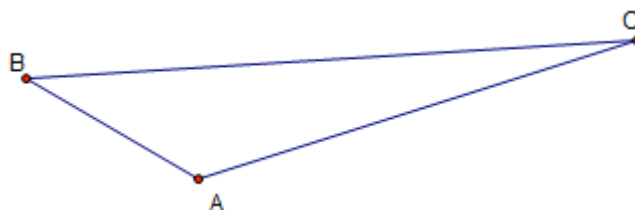
Q51. Would you prefer to introduce the geometrical topics you identified in the previous question first or have students solve a problem such as this one first? Explain.



## Section 6: Exploring a Triangle Center

In data analysis and statistics, measures of center that are commonly used are arithmetic means and medians. In statistics, the median is the middle value of an ordered data set. Medians, a segment between a vertex of a triangle and the midpoint of the opposite side, can be used to find a type of center of a triangle, called the **centroid**. Physically, the centroid of a triangle is its center of gravity. So if you cut out a triangle drawn on a uniformly dense piece of cardboard and found its centroid, ideally you would be able to balance the triangle on the top of a sharp object by placing the object at the centroid. There are other interesting properties of centroids that you will explore in this section.

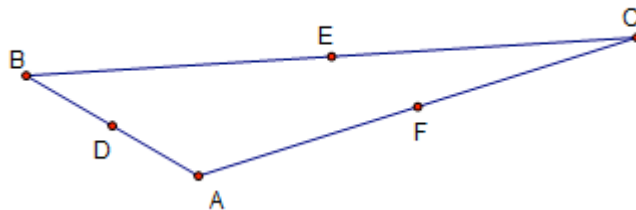
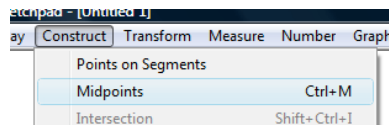
1. Begin by drawing a triangle by creating three points, labeled  $A$ ,  $B$ , and  $C$ , and  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$ .



2. Use your DGE to find the midpoints of each of the sides. To do this, choose



the selector tool, and click on the three segments. Select Midpoints from the Construct menu or use the Ctrl+M shortcut. Then, label the midpoint of  $\overline{AB}$ ,  $D$ , the midpoint of  $\overline{BC}$ ,  $E$ , and the midpoint of  $\overline{AC}$ ,  $F$ .



3. Draw  $\overline{BF}$ ,  $\overline{AE}$ , and  $\overline{CD}$ . What do you notice about these three segments when one of the vertices of the triangle is dragged?

From this simple sketch there are a variety of different relationships that could be investigated. Relationships may be noticed by looking for things that remain invariant, do not change, when objects in the sketch are changed by dragging. For example one might notice that a point remains fixed, two segments are always parallel, two values maintain the same ratio, or two polygons appear congruent or similar to each other. During this exploratory stage, it may be helpful to drag points,

measure the lengths of segments, find slopes of lines, or measure areas of polygons. After exploring, a **conjecture** is generally formulated. Conjectures come about from inductive reasoning; general statements being made based on a variety of suggestive examples. Those conjectures that turn out to be theorems are proved using deductive reasoning. This suggests that the user is probably convinced by data that has been collected empirically and supports a particular claim, but that claim has not been formally proven. Thus it is not always true unless shown otherwise through a mathematical proof.

### ENGAGING WITH CONTENT

Q52. Create two or more conjectures based on the diagram that you have created in your DGE. Be sure the conjecture is stated in “if-then” form.

Q53. Select one of the conjectures and construct a formal proof.

### CONSIDERATIONS FOR TEACHING

Q54. A student claims, “I have proved that the medians of a triangle always intersect in the interior of the triangle.” When you walk to his computer he demonstrates his proof by demonstrating to you that it is true “for all types of triangles.” He drags the triangle to make it obtuse, acute, right, scalene, isosceles, and equilateral. He states, “Because I have shown it to be true for all cases, it must always be true.” How do you respond?

## SUGGESTED ASSIGNMENTS

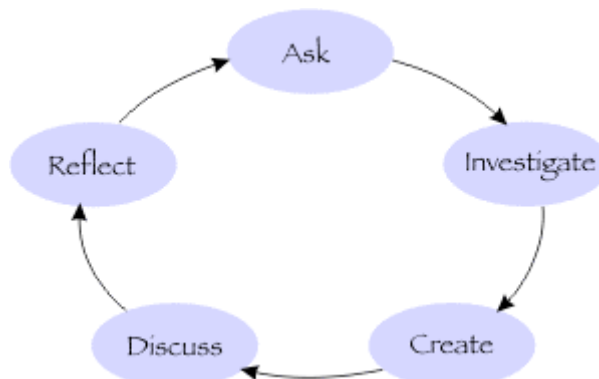
1. In Section One you created a list of important geometry topics for middle and high school students. Compare your list to the topics listed related to Geometry and Measurement in *The Curriculum Focal Points for Pre-Kindergarten through Grade 8 Mathematics* (NCTM, 2006). This document identifies important mathematical concepts that should be emphasized at each grade level and is intended to guide those who create mathematics curricula and state standards. If you have access to *Focus in High School Mathematics: Reasoning and Sense Making* (2009) you can also compare your list to those topics related to Geometry and Measurement suggested for high school students.

How does your list and the recommendations from the *Curriculum Focal Points* and *Focus in High School Mathematics* compare with your state's elementary and middle school standards?

2. In the module you learned how geometrical language can often confuse students because a single figure can have more than one classification. Sometimes students create their own language for describing a figure or portions of a figure and this can also cause confusion. In the videoclip, "Bottom.mov" you will observe two students working on a problem in which they are trying to create a triangle given three different side lengths and segments representing those side lengths. Play the videoclip and then respond to the following questions:
3. Students often ask, "Why do we need to know this?" because they are uncertain about how the topics they are learning are relevant to their lives. Real-world examples and applications are helpful in providing answers to questions such as this one. In this chapter you encountered a couple "real-world" problems. Find an additional real-world problem that is related to triangles. Describe the mathematical concepts that are addressed and, if appropriate, the ways technology could be used.
4. So far, you have constructed a right triangle and an isosceles triangle. Use your DGE to construct an equilateral triangle. Describe the method that you used to construct it. See if you can construct an equilateral triangle using another method.
5. When students are working in pairs or small groups to solve mathematics problems it is often helpful for teachers to provide directions to students about how they should work together. These directions might be different when

students are using technology. Consider the videoclip that you watched of students working on the triangle inequality theorem. What directions do you believe teachers should provide to students that can encourage effective group or pair problem solving?

6. One representation of an inquiry cycle is depicted in the picture below:



Source: <http://inquiry.illinois.edu/>

Describe how this cycle of inquiry can be specifically applied to learning geometry with the use of technology.

7. In the baseball stadium problem you constructed two different triangle centers: incenter and circumcenter. The centroid was also investigated in the section that followed. Search the internet to find out more about triangle centers and how they are related to each other, and also how some are useful for solving other real-world problems.

#### SUGGESTED READINGS:

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. *ZDM*, 34(3), 66-72.

Holz, R. (1996). How does 'dragging' affect the learning of geometry. *International Journal of Computers for Mathematical Learning*, 1(2), 169-187.

Lopez-Real, F., & Leung, A. (2006). Dragging as a Conceptual Tool in Dynamic Geometry Environments. *International Journal of Mathematical Education in Science & Technology*, 37(6), 665-679.

Talmon, V., & Yerushalmy, M. (2004). Understanding dynamic behavior: Parent-Child relations in dynamic geometry environments. *Educational Studies in Mathematics*, 57(1), 91-119.