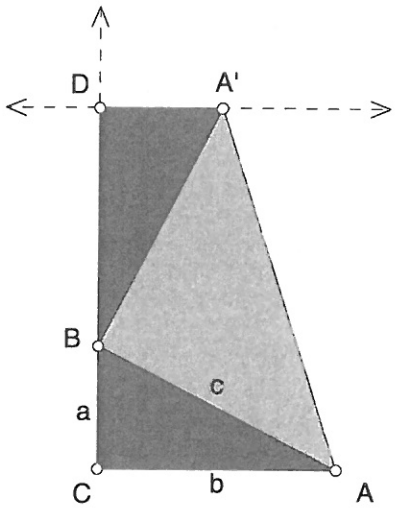
Presidential Pythagoras

Name(s): _____

James A. Garfield discovered a proof of the Pythagorean theorem in 1876, a few years before he became president of the United States. An interest in mathematics may not have been a prerequisite for the presidency, but it must have been common at the time. One of Garfield's predecessors, Abraham Lincoln, credited Euclid's *Elements* as being one of the books most influential to his career as a lawyer and politician, saying he learned from it how to think logically. Garfield's Pythagorean theorem proof is illustrated with a relatively simple figure: a trapezoid.

Sketch and Investigate

1. Construct a right triangle *ABC* and label it as shown.
2. Mark point *B* as center and rotate side *c* and point *A* by 90°.
3. Connect points *A* and *A´* and construct a line through point *A´*, parallel to side *b*.
4. Use the **Ray** tool to extend side *CB* and construct the point of intersection, *D*, of this ray and the line through *A´*.
5. Hide the ray and the line and replace them with segments *BD* and *DA´*.
6. Construct polygon interiors for the three right triangles.



Q1 What kind of figure is quadrilateral *ACDA´*? How do you know? Drag points *A*, *B*, and *C*. Does the figure remain this type of special quadrilateral?

Q2 What can you say about the triangles into which *ACDA´* is divided? Do they maintain these properties when you drag different parts of your sketch?

The point here is to use only side lengths and **Calculate** from the Measure menu to calculate areas. Don't actually measure areas until you're ready to confirm your calculations.

7. Measure sides *a*, *b*, and *c*. Now use just these measures to calculate the areas of the three triangles and their sum.
8. Now use the area formula for a trapezoid to calculate the area of *ACDA´* using just the side lengths. (What's the height of trapezoid *ACDA´*?) Construct the polygon interior of the entire figure and confirm your calculations were done correctly.

Prove

In steps 7 and 8, you calculated the area of the trapezoid in two different ways. Garfield used these two different ways of finding the area to prove the Pythagorean theorem. Can you do it too? Write two different expressions for the same area in terms of a, b, and c. Set these expressions equal and do the necessary algebra to arrive at the Pythagorean theorem.

Explore More

Look at the figure in the activity The Tilted Square Proof. How is Garfield's figure related to this one? See if you can transform (that's a hint) Garfield's figure into the Tilted Square figure. Compare the algebra involved in the two proofs and write a paragraph about how the proofs are related.