SKETCH AND INVESTIGATE

Q1 The exact length of the hypotenuse is $\sqrt{2}$ cm.

Q2 No. If the lengths of the legs are 1 cm and 2 cm, the hypotenuse does not have a whole-number length. Its length is $\sqrt{5}$ cm.

Q3 Students will need patience to complete all the rows in the table. If they are using a small screen, they may not find some of the bigger ones at the end of the list.

a	*b*	*c*
3	4	5
4	3	5
5	12	13
6	8	10
8	6	10
9	12	15
12	16	20
15	20	25

Q4 Any reordering of the three numbers of a triple shows a triangle congruent to the original. In the table, (3, 4, 5) and (4, 3, 5) are triples of congruent triangles, as are (6, 8, 10) and (8, 6, 10).

Q5 In the table, (3, 4, 5), (6, 8, 10), (9, 12, 15), and (15, 20, 25) represent triples of triangles similar to each other.

Q6 There are an infinite number of Pythagorean triples because you can multiply the numbers in any triple by the same positive integer to create another triple. For example, multiply (3, 4, 5) by 2 to get (6, 8, 10). Any triple that is not the integer multiple of another triple is called a *primitive Pythagorean triple.*

Q7 Students may check any three triples. For example,

$3^2 + 4^2 = 5^2$ because $9 + 16 = 25$

$5^2 + 12^2 = 13^2$ because $25 + 144 = 169$

$9^2 + 12^2 = 15^2$ because $81 + 144 = 225$

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EXPLORE MORE

11. When $m = 2$ and $n = 1$, Euclid's formula for Pythagorean triples gives the triple (3, 4, 5). Similarly, $m = 3$ and $n = 2$ generate the triple (5, 12, 13). You can use this formula to generate infinitely many Pythagorean triples because there are infinitely many choices for m and n. Euclid's formula generates every *primitive* Pythagorean triple, but it does not generate every triple that's a multiple of a primitive. For example, you cannot generate (9, 12, 15) using Euclid's formula because there are no whole-number values of m and n such that $m^2 + n^2 = 15$.

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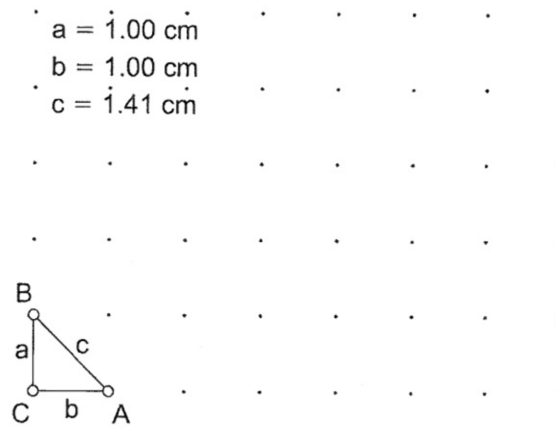
Pythagorean Triples

For GSP5

The Pythagorean theorem states that if a right triangle has side lengths a and b and hypotenuse length c, then $a^2 + b^2 = c^2$. A set of three whole numbers that satisfy the Pythagorean theorem is called a *Pythagorean triple.* In this activity you'll find as many right triangles as you can whose side lengths are whole numbers.

SKETCH AND INVESTIGATE



1. Make your sketch window as large as you can.

In the Edit menu, choose **Preferences** and go to the Units panel.

2. In Preferences, set the Distance Units to **cm** and the Distance Precision to **hundredths.**

In the Graph menu, choose **Show Grid;** then choose **Snap Points.** Select the grid (by clicking on a grid intersection) and choose **Display | Line Style | Dotted.**

3. Show the grid and turn on point snapping.
4. Hide the axes and the two control points.
5. In the lower-left corner of your sketch, draw a right triangle ABC with vertices on the grid.

Using the **Text** tool, click a segment to show its label. Double-click a label to change it.

6. Show the segment labels and change them to a, b, and c, as shown.
7. Measure the three side lengths.
8. Make the two leg lengths 1 cm each, as shown.

Q1 In this case, you can see that the hypotenuse length is not a whole number. Use the Pythagorean theorem to find the exact hypotenuse length (in radical form) when the side lengths are 1 cm. Show your work.

9. Drag point A one unit to the right.

Q2 When the leg lengths are 1 cm and 2 cm, is the hypotenuse length a whole number?

10. Drag point A one unit to the right again and look to see if the hypotenuse length is a whole number.

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You may or may not be able to fill in the whole chart, depending on the thoroughness of your search and the size of your screen. If your screen is very large, you may even need to add rows to the chart.

Q3 Continue a systematic search for Pythagorean triples, dragging point *A* one unit at a time to the right to increase *b* and dragging point *B* one unit up to increase *a*. Any time *c* is a whole number, record the Pythagorean triple in the chart at right.

a	*b*	*c*

Refer to your chart and experiment with the sketch to answer the following questions.

Q4 Which sets of triples are side lengths of congruent triangles?

Q5 Which sets of triples are side lengths of similar triangles (triangles with the same shape)?

Q6 Do you think there is a limit to the number of Pythagorean triples possible? Explain.

Q7 Use the Pythagorean theorem to verify at least three of your sets of triples and record your work.

EXPLORE MORE

11. Euclid's *Elements* demonstrates that Pythagorean triples can be generated by the formulas $m^2 - n^2$, $2mn$, $m^2 + n^2$, where m and n are positive integers and m is greater than n. What triple is generated by $m = 2$ and $n = 1$? Increase m and n and generate some other triples. Can you generate all the triples you recorded in your chart? Can you generate some triples that aren't on your chart? Draw some triangles with these side lengths on the grid to confirm that they're right triangles.

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