Triangle Sides and Sines: The Law of Sines

For GSP5

You have already used the sine, cosine, and tangent ratios to find missing parts of triangles. However, the definitions of these functions (involving the ratios of the opposite, adjacent, and hypotenuse) apply only to right triangles. In this activity you'll explore a different set of ratios that you can use in oblique triangles.

MORE RATIOS

1. In a new sketch, construct a triangle. Measure each angle and each side.

To show or change a label, select the object and then choose **Display | Show Label.**

2. The vertices are automatically labeled *A*, *B*, and *C*. Label the sides *a*, *b*, and *c* according to the vertex that is opposite each side.

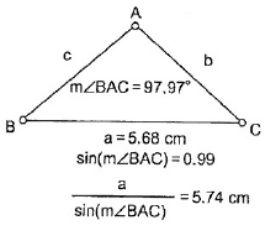
Q1 Drag the vertices to make $\angle A$ larger than $\angle B$. Which side is longer, *a* or *b*? Is this always true? Write down the measurements from three different examples.

Q2 Drag the vertices to make side *c* longer than *b*. Which angle is larger, $\angle B$ or $\angle C$?

To calculate the sine, choose **Number | Calculate.** Select **sin** from the Function pop-up menu, and then click the angle measurement in the sketch.

3. Calculate the sine of each angle.

4. Calculate the ratio of the length of each side to the sine of the opposite angle.



5. Select all three ratios and place them in a table by choosing **Number | Tabulate.** With the table still selected, choose **Number | Add Table Data** and choose to add ten entries as the values change.

Q3 Drag the vertices to change the angles and side lengths. What do you observe about the ratios?

Q4 Write your observation as an equation.

Q5 Calculate the reciprocal of each ratio. What do you observe? Write an equation.

These equations are both ways of writing the Law of Sines.

EXPLORE MORE

Q6 Open **Sides and Sines Proof.gsp.** Use the labels in the blue triangle to write a formula for sin *A*. Use the labels in the pink triangle to write a formula for sin *B*.

Q7 The length of segment *h* appears in both formulas. Solve both formulas for *h*, and set the results equal to each other.

Q8 What must you do to this equation to complete a proof of the Law of Sines?

Q9 By dragging point *C*, you can move segment *h* so it's outside $\triangle ABC$. Is your proof of the Law of Sines still correct, or must you modify it? Explain.