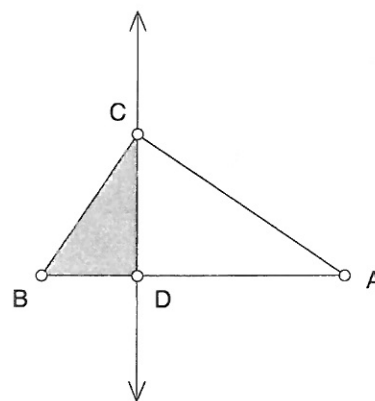


The Similar Triangle Proof Name(s): _____

The Pythagorean theorem proof most commonly found in geometry books is based on similar triangles. In this activity, though, you'll do more than simply prove the theorem. You'll puzzle out how similar triangles can be transformed into one another, and in the process you'll discover a surprising generalization of the Pythagorean theorem.

Sketch and Investigate

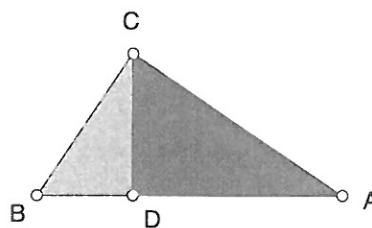
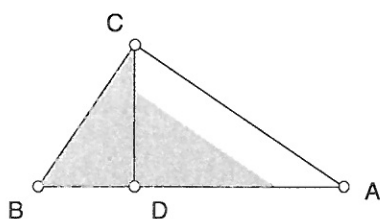
1. Construct a right triangle.
2. Construct a line through the right angle vertex C perpendicular to the hypotenuse.
3. Construct a segment CD , where D is the intersection of the perpendicular line and the hypotenuse. Then hide the line.
4. Construct the polygon interior of CBD .



Negative numbers
give clockwise
rotations.

- 5. Mark point D as center, select the interior of triangle BDC , and rotate it by -90° .

You should have a copy of $\triangle BDC$ positioned in the right angle corner of $\triangle CDA$. This rotated copy may make it easier to see a relationship between $\triangle BDC$ and $\triangle CDA$.



Mark Ratio and **Dilate** are both commands found in the Transform menu. Select two segments and choose **Mark Ratio**. Then select the figure you want to dilate and choose **Dilate**. Make sure **By Marked Ratio** is chosen in the Dilate dialog box.

- 6. Now you want to expand the rotated triangle to fill triangle CDA . To do this, select two segments whose ratio defines a scale factor that will enlarge the small triangle to fill $\triangle CDA$. Mark this ratio, and mark point D as a center for dilation. Dilate the rotated triangle by this marked ratio.

Q1 How are $\triangle BDC$ and $\triangle CDA$ related? Explain.

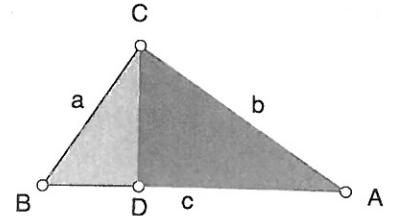
The Similar Triangle Proof (continued)

- Q2** Compare these two triangles to the whole triangle, BCA . Complete the following similarity statements:

Δ _____ $\sim \Delta$ _____ $\sim \Delta$ _____

7. Measure sides a and b and areas BDC and CDA . Calculate a^2 and b^2 , and compare these calculations to the areas.

- Q3** Write an equation relating a^2 , b^2 , and the two area measurements.



- Q4** Consider the following statements:

$$\text{area } \triangle BDC = ka^2 \text{ and area } \triangle CDA = kb^2.$$

Explain what these statements mean and why they are true based on your observations in step 6 and question Q3.

- Q5** Can a similar statement be made relating the area of $\triangle BCA$ to side c ? Measure and do calculations to confirm.

The investigation above highlights a very important feature of similar figures. By definition, corresponding lengths in similar figures are proportional. But the ratio of corresponding areas in similar figures is equal to the *square* of the ratio of corresponding lengths. So the areas of the similar triangles in your construction are proportional to the squares of their corresponding hypotenuses. The hypotenuses of the three triangles are a , b , and c . The area of the triangle with hypotenuse a can be written ka^2 , where k is some constant of proportionality.

- Q6** Clearly, $\text{area } \triangle BDC + \text{area } \triangle CDA = \text{area } \triangle BCA$. Write an equation relating a , b , c , and a proportionality constant, k . You've written a one-line proof of the Pythagorean theorem!

Explore More

The proof commonly found in geometry uses a proportion of corresponding sides in the similar triangles. Let BD in the figure above equal x . Write a proportion involving a , b , c , and x and simplify it to get the Pythagorean theorem.