

SKETCH AND INVESTIGATE

- Q1** As you drag point D , point F moves horizontally.
- Q2** As you drag point E around the circle, point F moves vertically up and down like a sewing-machine needle.
- Q3** Answers will vary. Students might sketch a path somewhat like the curve below.
- Q4** The sketch will look something like this. Also, if students leave the animation running, they will probably get a series of curves like this that will start to fill in the area around the curve.

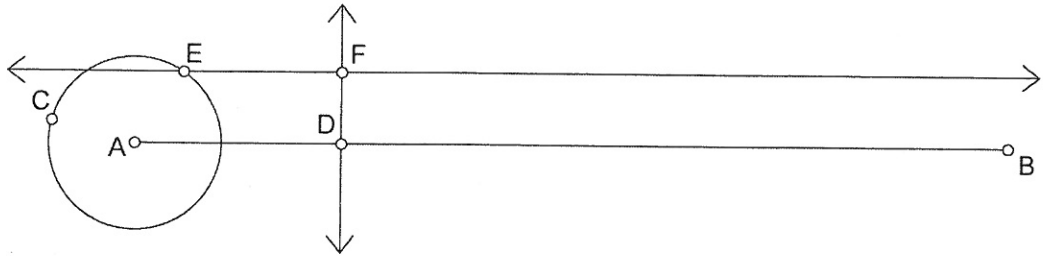


- Q5** The unit circle has a circumference of 2π , about 6.28 grid units.
- Q6** For the trace to repeat itself without tracing a new curve, the length of \overline{AB} must be an integer multiple of the circumference of the circle. The circumference of the circle is 2π , about 6.28 grid units, so the x -coordinate of point B should be about 6.28.

In this exploration you'll construct an animation "engine" that traces out a special curve called a *sine wave*. Variations of sine curves are the graphs of functions called *periodic functions*, functions that repeat themselves. The motion of a pendulum and ocean tides are examples of periodic functions.

SKETCH AND INVESTIGATE

1. Construct a horizontal segment AB .



2. Construct a circle with center A and radius endpoint C .
3. Construct point D on \overline{AB} .
4. Construct a line perpendicular to \overline{AB} through point D .
5. Construct point E on the circle.
6. Construct a line parallel to \overline{AB} through point E .
7. Construct point F , the point of intersection of the vertical line through point D and the horizontal line through point E .

Select point D and \overline{AB} ; then, in the Construct menu, choose **Perpendicular Line**.

Don't worry, this isn't a trick question!

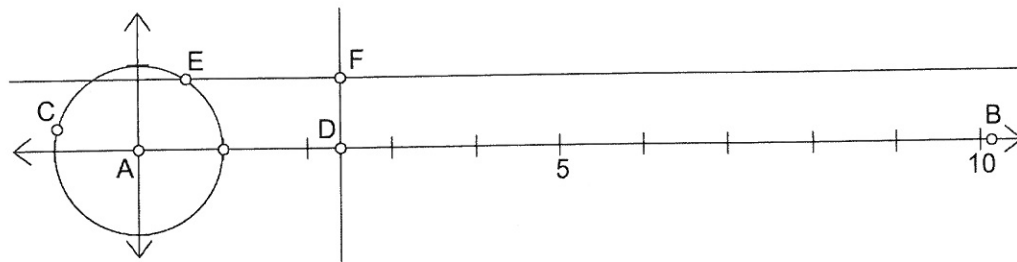
- Q1** Drag point D and describe what happens to point F .
- Q2** Drag point E around the circle and describe what point F does.
- Q3** In a minute, you'll create an animation in your sketch that combines these two motions. But first try to guess what the path of point F will be when point D moves to the right along the segment at the same time that point E is moving around the circle. Sketch the path you imagine.

Select points D and E and choose **Edit | Action Buttons | Animation**. Choose **forward** in the Direction pop-up menu for point D .

8. Make an action button that animates point D forward along \overline{AB} and point E forward around the circle.
9. Move point D so that it's just to the right of the circle.
10. Select point F ; then, in the Display menu, choose **Trace Point**.
11. Press the Animation button.

Q4 Sketch the path traced by point F . Does the actual path resemble your guess in Q3? How is it different?

12. Select the circle; then, in the Graph menu, choose **Define Unit Circle**. You should get a graph with the origin at point A . Point B should lie on the x -axis. The y -coordinate of point F above \overline{AB} is the value of the sine of $\angle EAD$.



Q5 If the circle has a radius of 1 grid unit, what is its circumference in grid units? (Calculate this yourself; don't use Sketchpad to measure it because Sketchpad will measure in inches or centimeters, not grid units.)

13. Measure the coordinates of point B .
14. Adjust the segment and the circle until you can make the curve trace back on itself instead of drawing a new curve every time. (Keep point B on the x -axis.)
- Q6** What's the relationship between the x -coordinate of point B and the circumference of the circle (in grid units)? Explain why you think this is so.