Solving Linear Equations by Balancing

To solve a complicated equation, you can make it simpler while keeping the two sides of the equation equal. In this activity you will use a Sketchpad balance to show an equation and solve it by using operations that keep the sides balanced.

EXPLORE THE BALANCE

1. Open **Solve by Balancing.gsp.** Experiment by dragging objects from the storage area (on the left of the dividing bar) to the left or right balance pan.

Q1 Which objects weigh a pan down, and which ones pull it up?

Q2 Find a combination of weights and balloons different from the one shown below that balances the two pans. List the objects you put on each pan.

Q3 Write down the algebraic equation that corresponds to your arrangement of weights and balloons. Press the *Show Formula* button to check your answer.

When the pans are balanced, some operations disturb the balance and others do not.

Q4 Try each of these operations and write down how it affects the balance. Before each operation, press *Setup Q4* to make sure the pans are balanced and that each pan contains enough items to carry out that operation.

In your answer, state whether the operation makes the left pan heavier, makes the right pan heavier, or leaves the two pans in balance.

a. Drag a 1 from the storage area onto each pan.

b. Drag a 1 from the right pan to the left pan.

c. Drag a 1 and a −1 together from the left pan to the right pan.

d. Drag a 1 from the storage area onto the left pan and a −1 onto the right pan.

e. Drag a −5 onto each pan.

f. Remove an x from each pan by dragging to the storage area.

g. From the storage area drag an x onto the left pan and a 5 onto the right pan.

h. Drag an x and a $-x$ from the right pan to the storage area.

Q5 Write down at least two rules to describe things you can do that will keep the pans balanced. For each rule, write down which parts of Q4 illustrate the rule.

SOLVE AN EQUATION

Q6 Go to page 2. What equation does this balance represent? Press the *Show Formula* button to check your answer.

In the next few steps you will use these balancing rules:

Rule 1: You can drag the same kind of object from the storage area to each of the pans. (For example, you can drag a 5 to the left pan and a 5 to the right pan.)

Rule 2: If you have matching positive and negative objects on the same pan, you can remove them to the storage area. (For instance, if the left pan has both a 5 and a -5, you can remove them both.)

Make sure the pans stay in balance after each operation.

Q7 Drag a $-x$ from the storage area onto each pan. Which rule allows you to do this?

Q8 Any time you find both an x and a $-x$ on the same pan, you can remove them to the storage area. Which rule is this? How many such combinations can you find? Remove them now.

Q9 What is the resulting equation?

Q10 Drag a 1 from the storage area onto each pan. Any time you find both a 1 and a -1 on the same pan, you can remove them to the storage area. How many such combinations can you find and remove?

Q11 What is the resulting value of x? Press the *Show x* button to check your result.

MORE EQUATIONS

You have solved the equation when you have x by itself on one pan and only numbers on the other pan.

2. On page 3, use the rules about moving objects to eliminate as many objects as you can and to leave the last x all alone on its pan.

Q12 Write down each step that you follow, and write down the equation for the balance after each step.

Q13 On page 4, what equation does the balance show?

Q14 Use the two rules to add and remove objects until you get x on a pan by itself, with only numbers on the other pan. Each time you use Rule 1, write down what you did and the resulting equation. What is x?

Q15 Remove all the objects from the pans by pressing the *Reset* button, and then move a single x onto the left pan. Does this object weigh the pan down or pull it up? Is your answer the same as it was for Q1? If not, why not?

Q16 Go to page 5 and add objects to the pans to model the equation $4x - 2 - x = x + 3 + x$. Then solve the equation by following the two rules.

EXPLORE MORE

Q17 There's one more rule you can use with the algebra balance. Go to page 6 and notice that each pan has two identical piles of objects. Remove one pile from the left pan and one pile from the right pan. Do the pans stay in balance? What mathematical operation did you perform on each pan?

Q18 With the help of this third rule, solve these three equations:

a. $2x - 1 = 5$ b. $4x + 3 = 2x - 3$ c. $4x + 5 = x - 7$

Q19 Page 7 contains several buttons to create different arrangements of objects. For each button, press the button and write down the equation that results. Then use the various rules you've learned to rearrange the objects and solve the equation, and write down the solution that you find.

Q20 Page 8 is a blank page. Show the value of x, adjust the slider, and arrange objects to make a balanced equation of your own choosing. Then hide the x value and slider, and challenge a friend to solve your equation.

Use Sketchpad's Help menu to find out how to make and use movement buttons.

Q21 Page 9 is also a blank page. Make a movement button to move objects out of the storage area and onto the pans to make an equation. Then make a movement button for each step in solving the equation. Use the movement buttons to demonstrate your problem to your classmates.