Subtracting Integers

Activity Notes

Objective: Students use an animated Sketchpad model for subtracting integers on the number line, and see the second number being flipped before it's added to the first number. Students investigate subtraction of two positive numbers and various subtraction problems involving negative numbers.

Student Audience: Pre-algebra/Algebra 1

Prerequisites: None. This will be review for most Algebra 1 students.

Sketchpad Level: Easy. Students manipulate a prepared sketch.

Activity Time: 20–30 minutes. You may want to combine this activity and the Adding Integers activity in a single class period.

Setting: Paired/Individual Activity (use **Subtracting Integers.gsp**) or Whole-Class Presentation (use **Subtracting Integers Present.gsp**)

Use this activity as an introduction to integer subtraction for pre-algebra students, as a start-of-the-year refresher for Algebra 1 students, or as a supplemental activity for any student having difficulty with the topic. It's important for students to have a mental image of operations on integers. Even strong students who rely on verbal rules make careless mistakes that could be avoided by having an internalized picture.

The picture of subtraction presented here is a geometric model in which each number is represented by a vector. (The activity calls them *arrows* because students may not be familiar with the term *vector*.) Vectors incorporate both magnitude and direction (representing the absolute value and the sign of the integer), so practice with this model helps students understand how the signs of the operands come into play.

The questions are critical in encouraging students to internalize the model presented in this activity. Make sure students write clear and detailed explanations (and use complete sentences) when they answer the questions; the extra time it takes them to do so is time well spent.

If there's time and you have a presentation computer with a projector, have different students use Sketchpad to demonstrate to the class their observations or the problems they made up. It's a big help to students if they can listen to, evaluate, and discuss the descriptions and conclusions of their classmates.

INVESTIGATE

These notes sometimes use the terms *minuend* (first number) and *subtrahend* (second number), but these terms are not used in the student material. If you do use them with students, be sure to explain them carefully.

The concept of *additive inverse* is not named, but it plays a prominent role in the animation. You should discuss with the class why the second number must be flipped, even if you don't give a name to that operation.

Q1 During the animation, the arrow for 5 flips from the right to the left. This shows which way the second arrow must go in order to subtract it from the first.

Q2 In their final positions, the flipped second arrow starts from where the first arrow ends, and the answer (3) is at the end of the second arrow. Encourage students to be detailed and specific in their answer to this question.

Q3 Answers will vary. Students should describe the arrow flipping from right to left; encourage them to explain in their own words why it needs to flip in order to do subtraction.

Q4 Answers will vary but should include only problems in which a positive minuend is smaller than a positive subtrahend.

Q5 If both numbers are positive, the result will be positive if the first number (minuend) is larger, and negative if the second number (subtrahend) is larger.

Q6 Some students will record direct observations, and others will interpret those observations. Typical answers will be similar to the following.

Observation: In this problem, $4 - (-3)$, the second arrow starts out pointing to the left, so when it flips it turns around and points to the right.

Interpretation: The second number starts out negative, so when it flips it becomes positive.

Q7 The problems students create will vary. Because the first number is positive and the second negative, the

models have in common that, after flipping, both arrows point to the right, and the result must be positive.

Q8 Problems will vary. Because the first number is negative and the second positive, after flipping, both arrows point to the left, and the result is negative.

Q9 As students model various problems, walk around the room and observe them to make sure they can model any problem they are given.

$7 - (-4) = 11$ $\quad -4 - 7 = -11$

$-6 - (-2) = -4$ $\quad -3 - (-6) = 3$

$-3 - 8 = -11$ $\quad -3 - (-8) = 5$

$2 - (-7) = 9$ $\quad -2 - 7 = -9$

Q10 Written as addition problems, these problems become

$7 + 4 = 11$ $\quad -4 + (-7) = -11$

$-6 + 2 = -4$ $\quad -3 + 6 = 3$

$-3 + (-8) = -11$ $\quad -3 + 8 = 5$

$2 + 7 = 9$ $\quad -2 + (-7) = -9$

In each case, to subtract you can change the sign of the second number and add them. This is similar to the way the second arrow flips before the animation shows the answer.

EXPLORE MORE

Q11 For a subtraction problem to have an answer of zero, the two numbers being subtracted must be the same.

Q12 To make the difference the same as the first number, the second number must be zero.

Q13 To make the difference the same as the second number, the first number must be twice as big as the second. For instance, $6 - 3 = 3$, and $-8 - (-4) = -4$.

Q14 The order does matter when you subtract numbers, because only the second arrow is flipped. More sophisticated students will observe that the order matters only if the second number is nonzero, because flipping zero has no effect.

WHOLE-CLASS PRESENTATION

Start the whole-class presentation by animating the subtraction of two positive integers (Q1–Q5 of the activity). Open the sketch **Subtracting Integers Present.gsp** and press the step-by-step buttons one at a time, pausing between animations. Ask students to describe what they see as the animation progresses, and be sure to get observations from several different students. Press the *Reset* button, change the problem by dragging both circles (while leaving the numbers positive), and press the step-by-step buttons again. Pay special attention to Q3 and Q5.

Next animate subtraction problems in which the first number is positive and the second number is negative (Q6–Q7 of the activity). Press *Reset,* make the first number positive and the second negative, and ask students to predict what will happen now. Test their conjectures using the step-by-step buttons. Repeat for several more problems.

Animate subtraction problems like those in Q8 and Q9, and record the answers for each of the problems in Q9. Ask students what patterns they see, and how they could predict the answer from the two numbers being subtracted.

For Q10, ask students to make an addition problem for each of the problems from Q9, and test their addition problems using page 2 of the sketch. Switching back and forth between page 1 and page 2 will reinforce for students the idea of using addition to rewrite a subtraction problem.

Continue the class discussion with as many of the Explore More questions (Q11–Q14) as are appropriate for the class and the available time.

Finish by having students summarize in their own words the relationship between subtraction and addition.