

SKETCH AND INVESTIGATE

Q1 When the angles change, the ratios among the triangle's side lengths also change.

Q2 When the triangle changes scale without changing shape, the ratios among the triangle's sides don't change. The different triangles produced by dragging A or B are similar to each other, so their sides remain proportional.

Q3

$$\text{sine of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{cosine of } \angle A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\text{tangent of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

Q4 $\sin 30^\circ = 0.500$, $\cos 30^\circ = 0.866$, $\tan 30^\circ = 0.577$

Q5 The measure of $\angle C$ will vary depending on the measure of $\angle A$, but the two will always sum to 90° . The sine of $\angle C$ equals the cosine of $\angle A$. This is because the side opposite $\angle C$, side AB , is the side adjacent to $\angle A$.

- Q6**
- The sine of an angle of 0° has value 0. This is the smallest value possible for the sine of an angle in a right triangle. (The sine of angles with negative values can get as small as -1 . But negative angles cannot exist in a right triangle.)
 - The greatest possible value for the sine of an angle is 1. The angle with this sine is 90° . Although students won't be able to make $\angle A$ 90° in the sketch, they can see the sine approaching 1 as $\angle A$ approaches 90° .
 - If $\angle BAC = 90^\circ$, the triangle has two right angles, which is impossible.
 - If you drag point C up as much as possible, the measure of $\angle A$ approaches 90° . At the same time, the length of the side opposite $\angle A$ gets larger and larger, while the length of the side adjacent to $\angle A$ stays the same. Since the tangent is the ratio of opposite/adjacent, the tangent ratio gets extremely large, approaching infinity as $\angle A$ approaches 90° .

Trigonometric Ratios

continued

For
GSP5

ACTIVITY NOTES

- e. The tangent equals 1 when the ratio of opposite/adjacent equals 1. This is true when the triangle is isosceles, which happens when $\angle A$ measures 45° .
- f. The cosine and sine are equal when the triangle is isosceles. This means $\angle A$ measures 45° . The two ratios are equal because the sides opposite and adjacent to $\angle A$ are the same length.
- g. $\sin x = \cos(90 - x)$

Trigonometric Ratios



Right-triangle trigonometry builds on similar-triangle concepts to give you more ways to find unknown measures in triangles. In this activity you'll learn about trigonometric ratios and how you can use them.

SKETCH AND INVESTIGATE

In steps 1–5, you'll construct a right triangle.

Select point B and \overline{AB} ; then, in the Construct menu, choose **Perpendicular Line**.

Using the **Text** tool, click a segment to show its label. Double-click the label to change it.

Select, in order, points C , A , and B . Then, in the Measure menu, choose **Angle**.

For each ratio, select the two segments in order. Then, in the Measure menu, choose **Ratio**.

To measure an angle, press the **Marker** tool on the vertex and drag into the angle. Then select the angle marker and choose **Measure | Angle**. To measure a distance, select the two points and choose **Measure | Distance**.

1. Construct \overline{AB} .

2. Construct a line through point B perpendicular to \overline{AB} .

3. Construct \overline{AC} , where point C is a point on the perpendicular line.

4. Hide the line.

5. Construct \overline{BC} to finish the right triangle.

6. Show the three segments' labels and change the labels to match the figure above right.

7. Measure $\angle CAB$.

8. Measure the ratios *opposite/hypotenuse*, *adjacent/hypotenuse*, and *opposite/adjacent*.

Q1 Drag point C to change the angles. When the angles change, do the ratios also change?

Q2 Drag point A or point B to scale the triangle. What do you notice about the ratios when the angles don't change? Explain why you think this happens.

Your observations in Q2 give you a useful fact about right triangles. For any right triangle with a given acute angle, each ratio of side lengths has a given value, regardless of the size of the triangle. The three ratios you measured are called *sine*, *cosine*, and *tangent*.

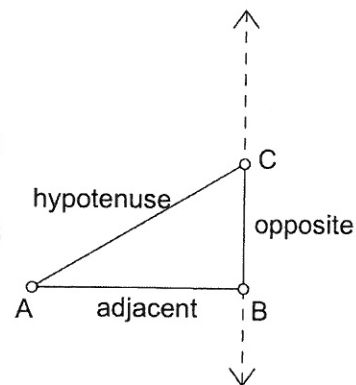
9. The sine, cosine, and tangent functions can be found on all scientific calculators, commonly abbreviated as sin, cos, and tan. Use Sketchpad's Calculator to calculate the sine, cosine, and tangent of $\angle CAB$. Match these calculations with the ratios they are equal to.

$$m\angle CAB = 31^\circ$$

$$\frac{m \text{ opposite}}{m \text{ hypotenuse}} = 0.51$$

$$\frac{m \text{ adjacent}}{m \text{ hypotenuse}} = 0.86$$

$$\frac{m \text{ opposite}}{m \text{ adjacent}} = 0.60$$



Q3 Complete the ratios for cosine and tangent.

$$\text{sine } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{cosine } \angle A = \underline{\hspace{2cm}}$$

$$\text{tangent } \angle A = \underline{\hspace{2cm}}$$

Q4 Drag point C so that $\angle A$ measures as close to 30° as you can get it. Write approximate values for the sine, cosine, and tangent of 30° below. Use the definitions in Q3 and refer to the calculations in your sketch to find these values.

$$\sin 30^\circ = \underline{\hspace{2cm}} \quad \cos 30^\circ = \underline{\hspace{2cm}} \quad \tan 30^\circ = \underline{\hspace{2cm}}$$

Q5 Without measuring, figure out the measure of $\angle C$ and write down that number. Calculate the sine of that angle measure. The sine of $\angle C$ should be close to one of the trigonometric ratios for $\angle A$. Which one? Explain why this is so.

Q6 Drag point C and answer the following questions.

- What's the smallest possible value for the sine of an angle in a right triangle? What angle has this value?
- What's the greatest possible value for the sine of an angle in a right triangle? What angle has this value?
- Why can't you make $m\angle CAB$ exactly equal to 90° ?
- Even though you can't make $m\angle CAB$ exactly equal to 90° , what do you think is the value of $\tan 90^\circ$? Explain.
- For what angle is the tangent equal to 1? Why?
- For what angle are the sine and cosine equal? Why?
- Suppose an angle has measure x . Complete this equation:

$$\sin x = \cos \underline{\hspace{2cm}}$$

Hint: Make \overline{AB} short so that you can drag point C up farther.