General Form Quadratic—Escape Ramp

On highways through mountainous regions, engineers often construct emergency escape ramps. Vehicles that have lost their brakes can use these ramps to come to a safe stop. To design one of these ramps, the engineers ran some tests with a truck traveling at various speeds. Your goal is to use their test data to figure out how long one of these ramps should be.

Q1 What factors do you think the engineers need to take into account?

INVESTIGATE

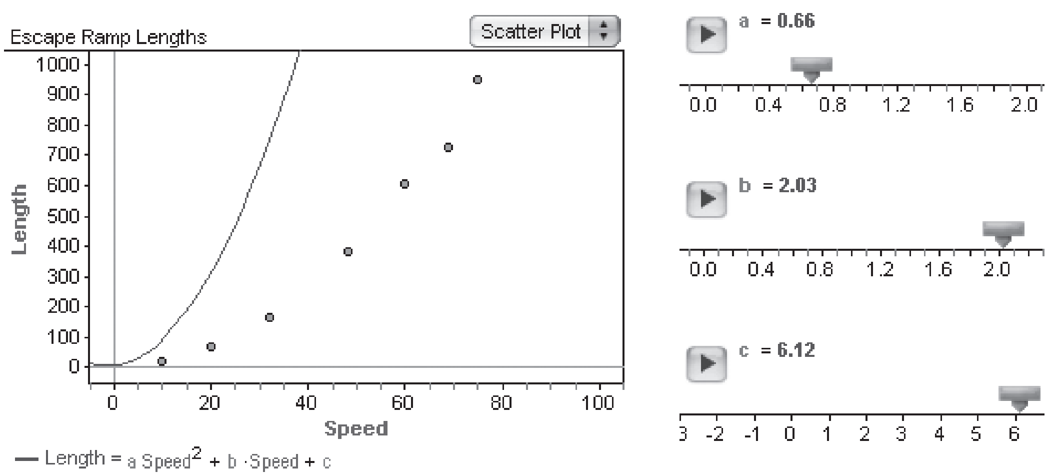
1. Open the Fathom document **Escape Ramp.ftm.** You will find a set of data that includes the speed of the truck, in miles per hour, at the start of the ramp and the distance to stop without using brakes, in feet.

For these data points, the engineers were using a ramp covered with sand and with a grade of 6% (that is, a slope of $\frac{6}{100}$). You find out that a ramp should be designed for a speed of 90 mph. The engineers didn't make a test at that high a speed, so you need to make a prediction.

2. Start by creating a scatter plot of the data, with *Speed* on the horizontal axis.

Because the data points appear to be curved, they might be fit by the graph of a quadratic function: $f(Speed) = a \cdot Speed^2 + b \cdot Speed + c$.

3. Drag down sliders for the coefficients *a, b,* and *c.* Plot the function with formula a • Speed² + b • Speed + c. To see the parabola, set the sliders near the position shown here.



4. Because you need to make a prediction for a faster speed, enlarge your graph window to include speeds of at least 90 mph, as well as some negative values.

Q2 As you slide the value of *c*, how does the graph change? Include negative values of *c*. Explain your observations. Consider the location of the vertex and the shape of the parabola, as well as whatever else you see.

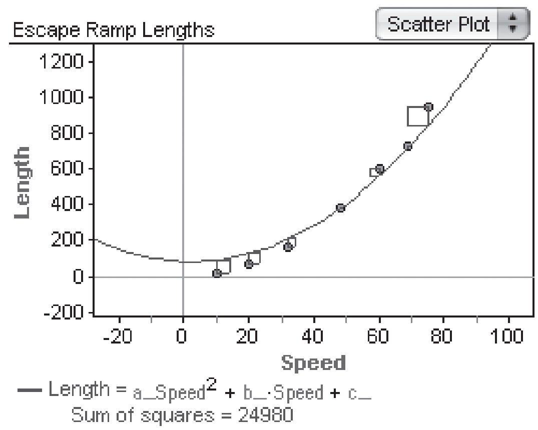
Q3 As you slide the value of *b* to positive and negative values, how does the graph change? Again, consider the vertex and the shape, among other things.

Q4 As you slide the value of *a*, how does the graph change?

Once you adjust the sliders so that the function graph fits the data points, you can determine the equation from the slider values.

Q5 What values of *a, b,* and *c* give a graph that fits the data points closely? You might choose **Graph | Show Squares** to help make a good fit.

Q6 How long should a ramp be to handle vehicles traveling at 90 mph?



EXPLORE MORE

1. A quadratic function can also have the form $f(x) = a(x - h)^2 + k$. In this form, you can tell that the parent function $f(x) = x^2$ has been shifted *h* units horizontally and *k* units vertically and has been stretched or shrunk vertically by a factor of *a*. Use *c* to represent $f(x)$; algebraically write *k* in terms of *a*, *c*, and *h*. If *a* is fixed and you keep the value of $f(x)$ the same for some particular value of *x* (that is, you also fix *x* and *c*) while moving the slider for *h*, what is the path of the vertex?

2. Add units to the attributes in the table. You'll get error messages about incompatible units on the function or functions being graphed. To clear up these error messages, go to the sliders and enter appropriate units after the values.

   Fathom automatically changes ft/mph to s. Explain the relationship between ft/mph and seconds.

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On highways through mountainous regions, engineers often construct emergency escape ramps. Vehicles that have lost their brakes can use these ramps to come to a safe stop. To design one of these ramps, the engineers ran some tests with a truck traveling at various speeds. You can find their data in **Escape Ramp.ftm.** Unfortunately, you need to plan a ramp for a speed they didn't test: 90 mph. Use sliders for the coefficients of a quadratic function to fit their data and decide the length of a ramp to handle a runaway speed of 90 mph.