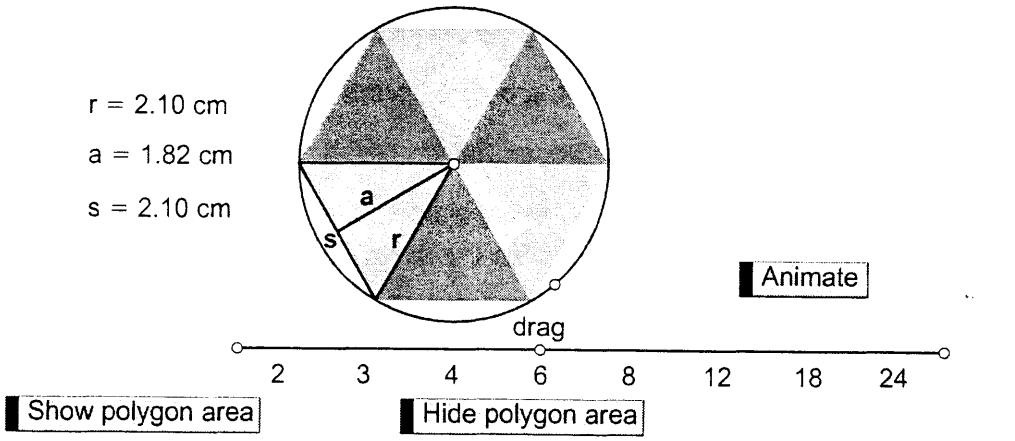
Areas of Regular Polygons and Circles

For GSP5

A regular polygon has congruent sides and congruent angles. You can divide any regular polygon into congruent triangles by drawing segments from the center to each vertex. You can then use the area of one of these triangles to find the area of the polygon. This method can be extended to derive a formula for the area of a circle. In this activity you'll manipulate a sketch in which you can vary the number of sides of a regular polygon to see how that affects the area.

SKETCH AND INVESTIGATE

1. Open the sketch **To a Circle.gsp.**



2. Drag (or animate) point *drag* along its segment. Observe how the polygon and the measurements change.

3. Drag to give the polygon four or more sides.

Q1 Each regular polygon with four or more sides is divided into triangles. Look at the triangle with a thick outline. It has segments labeled *a* for *apothem* and *s* for *side.* (The *apothem* of a regular polygon is the perpendicular distance from the center to one of the sides.) Write a formula for the area of the triangle using *a* and *s*.

Choose **Calculate** from the **Number** menu to open the Calculator. Click a measurement to enter it into a calculation.

Q2 Use the measurements for *a* and *s* to calculate an expression for the area of the polygon. Press the *Show polygon area* button to confirm that you've made the correct expression. Record your expression below. *Note:* Your calculation will disappear if you change the number of sides of the polygon.

Q3 Change the number of sides of the polygon and calculate a new expression for its area. Check this new expression against the area given in the sketch. Record this expression.

Q4 If you've successfully calculated expressions for a couple polygons, you should be ready to write a general formula for the area of a regular polygon. Write a formula for area A using a for apothem, s for side length, and n for number of sides.

Q5 Write a formula for the perimeter p of a regular polygon with n sides and with side length s.

Q6 Rewrite your formula in Q4 using p instead of s and n.

4. With the polygon area showing, drag or animate point *drag* some more, focusing on what happens to the polygon as the number of sides increases.

Q7 What happens to the polygon as the number of sides increases?

Q8 What does the apothem approach as the number of sides increases?

Q9 What does the perimeter approach as the number of sides increases?

5. Use the measurements for circumference and for r (radius) to calculate an expression equal to the area of the circle.

Q10 Write a formula for the area of a circle A using C for circumference and r for radius.

Q11 The formula for circumference is $C = 2\pi r$. Substitute $2\pi r$ for C in your formula in Q10 and simplify.

SKETCH AND INVESTIGATE

Q1 $as/2$ = area of the triangle

Q2 and **Q3** Answers will vary. For a four-sided polygon, the expression should resemble $4(as/2)$.

Q4 $A = (as/2)n$ or $A = (1/2)(asn)$

Q5 $p = sn$

Q6 $A = (1/2)(ap)$

Q7 As the number of sides increases, the polygon becomes more and more like a circle.

Q8 As the number of sides increases, the apothem becomes more and more like the radius of the circle.

Q9 As the number of sides increases, the perimeter approaches the circumference of the circle.

Q10 $A = (1/2)Cr$

Q11 $A = (1/2)(2\pi r)(r)$

$A = \pi r^2$