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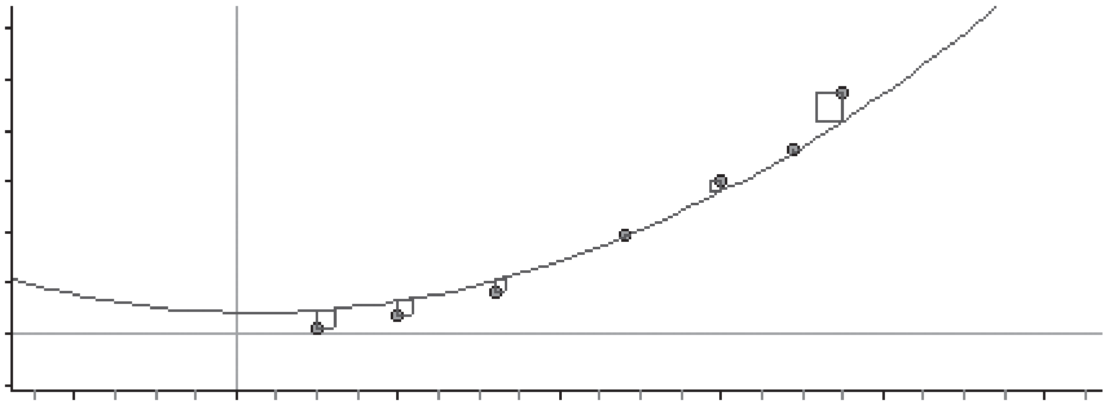
Investigating Higher-Degree Polynomials



General Form Quadratic—Escape Ramp

On highways through mountainous regions, engineers often construct emergency escape ramps. Vehicles that have lost their brakes can use these ramps to come to a safe stop. To design one of these ramps, the engineers ran some tests with a truck traveling at various speeds. Your goal is to use their test data to figure out how long one of these ramps should be.

Q1 What factors do you think the engineers need to take into account?

INVESTIGATE

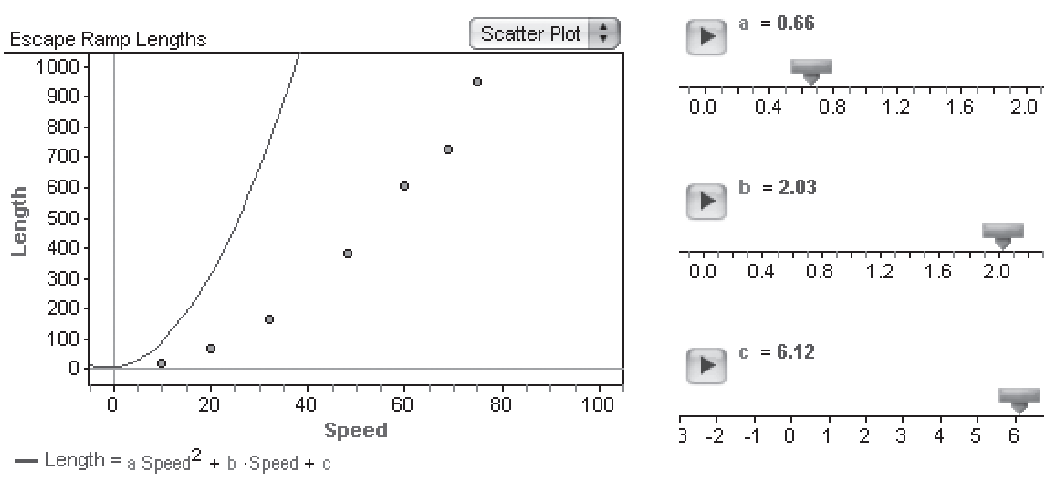
1. Open the Fathom document **Escape Ramp.ftm.** You will find a set of data that includes the speed of the truck, in miles per hour, at the start of the ramp and the distance to stop without using brakes, in feet.

For these data points, the engineers were using a ramp covered with sand and with a grade of 6% (that is, a slope of $\frac{6}{100}$). You find out that a ramp should be designed for a speed of 90 mph. The engineers didn't make a test at that high a speed, so you need to make a prediction.

2. Start by creating a scatter plot of the data, with *Speed* on the horizontal axis.

Because the data points appear to be curved, they might be fit by the graph of a quadratic function: $f(Speed) = a \cdot Speed^2 + b \cdot Speed + c$.

3. Drag down sliders for the coefficients *a*, *b*, and *c*. Plot the function with formula a • Speed² + b • Speed + c. To see the parabola, set the sliders near the position shown here.



4. Because you need to make a prediction for a faster speed, enlarge your graph window to include speeds of at least 90 mph, as well as some negative values.

Q2 As you slide the value of c, how does the graph change? Include negative values of c. Explain your observations. Consider the location of the vertex and the shape of the parabola, as well as whatever else you see.

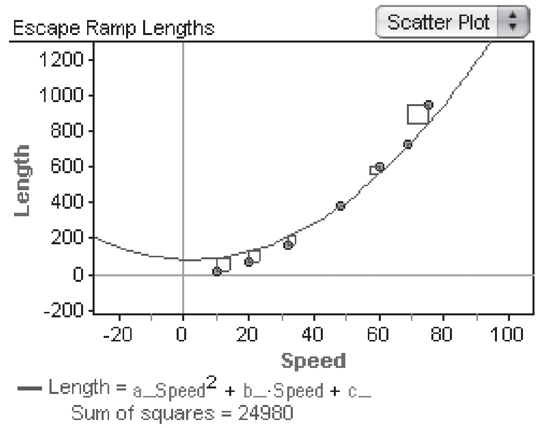
Q3 As you slide the value of b to positive and negative values, how does the graph change? Again, consider the vertex and the shape, among other things.

Q4 As you slide the value of a, how does the graph change?

Once you adjust the sliders so that the function graph fits the data points, you can determine the equation from the slider values.

Q5 What values of a, b, and c give a graph that fits the data points closely? You might choose **Graph | Show Squares** to help make a good fit.

Q6 How long should a ramp be to handle vehicles traveling at 90 mph?



EXPLORE MORE

1. A quadratic function can also have the form $f(x) = a(x - h)^2 + k$. In this form, you can tell that the parent function $f(x) = x^2$ has been shifted h units horizontally and k units vertically and has been stretched or shrunk vertically by a factor of a. Use c to represent $f(x)$; algebraically write k in terms of a, c, and h. If a is fixed and you keep the value of $f(x)$ the same for some particular value of x (that is, you also fix x and c) while moving the slider for h, what is the path of the vertex?

2. Add units to the attributes in the table. You'll get error messages about incompatible units on the function or functions being graphed. To clear up these error messages, go to the sliders and enter appropriate units after the values.

   Fathom automatically changes ft/mph to s. Explain the relationship between ft/mph and seconds.

General Form Quadratic—Escape Ramp

On highways through mountainous regions, engineers often construct emergency escape ramps. Vehicles that have lost their brakes can use these ramps to come to a safe stop. To design one of these ramps, the engineers ran some tests with a truck traveling at various speeds. You can find their data in **Escape Ramp.ftm.** Unfortunately, you need to plan a ramp for a speed they didn't test: 90 mph. Use sliders for the coefficients of a quadratic function to fit their data and decide the length of a ramp to handle a runaway speed of 90 mph.

Objective: Students will use Fathom and a real-world problem to dynamically explore how a graph is affected by the coefficients in the general form of a quadratic.

Student Audience: Algebra 1, Algebra 2

Activity Time: 25–40 minutes

Setting: Paired/Individual Activity, Exploration, or Whole-Class Presentation (use **Escape Ramp.ftm** for any setting)

Mathematics Prerequisites: Students understand points in a scatter plot, equation of a curve, and fitting a function to data in order to make a prediction.

Fathom Prerequisites: Students can make a scatter plot and adjust the graph window, plot and trace a function, create and name sliders, and show residual squares.

Notes: As you listen to students describe what they see in Q2–Q4, encourage complete descriptions by asking about the shape, even when it is not changing. Ask about the y-intercept if students don't mention it as they describe what they see as slider a is moved. Ask what happens when $a = 0$. If students see little change, encourage them to enlarge the scale on their sliders to see big changes. As students work on Q5, encourage them to set windows similar to those shown in step 3. Sliders in dynamic data software allow students to visualize the path followed by the parabola's vertex as coefficients change. The visualization can provoke the question "Why?"

For an Exploration: Students familiar with Fathom who do not need step-by-step guidance can use the Exploration. Their model and the approximation they make from their model will be close to answers for Q5 and Q6. As they refine their model, you might suggest the tool **Show Squares.**

For a Presentation: As you build the Fathom document from data, start with wide scales for the slider parameters so students can see clear differences as the sliders are moved. Draw out complete descriptions, letting several students contribute to those for Q2–Q4. For Q5, set the scales to measure in hundredths. As you **Show Squares**, ask students to describe what the squares are. Include Explore More 1 in the presentation. As you discuss Q6, you might ask, Is it reasonable to extend the linear data to fit trucks traveling at 90 mph? Students will have various opinions. The engineers planning the ramp will need to know the maximum speed for which the linear relationship seen in the tests holds.

Q1 Answers will vary. Factors include the speed and weight of the runaway vehicle, the grade of the ramp, and the material from which the ramp is constructed.

INVESTIGATE

Q2 The parabola shifts up and down, as seen by the y-intercept (probably close to the vertex). The shape of the parabola does not change.

Q3 The vertex follows a path of a parabola opening down. The shape of the parabola remains the same.

Q4 The parabola closes and opens, corresponding to vertical stretches and shrinks (which are horizontal shrinks and stretches, respectively) as the absolute value of a increases and decreases. A negative value of a inverts the parabola, making it open downward. The y-intercept stays the same.

Q5 Answers will vary. Theoretically, the best quadratic fit is $Distance = 0.16Speed^2 - 0.09Speed + 2.45$. Student answers may vary more on values for b and c, which have less effect on the parabola through these data points. Ask whether it makes sense for the vertex to be at the origin. Of graphs with vertices at the origin, good fits have equations close to $Distance = 0.15Speed^2$.

Q6 Answers will vary. The models in Q5 give lengths of 1290 and 1215 ft, respectively.

EXPLORE MORE

1. $k = c - a(x - h)^2$. If a and c are fixed, the equation is a quadratic function of h, so the path is parabolic.
2. Slider a is in $\frac{\text{ft}}{\text{mph}^2}$, b is in $\frac{\text{ft}}{\text{mph}}$, and c is in ft.

$$1 \cdot \frac{\text{ft}}{\text{mph}} = \text{ft} \cdot \frac{\text{hr}}{\text{mi}} \cdot \frac{\text{mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ s}}{\text{hr}}$$
$$= 0.682 \text{ s}$$

Units were not used in the main part of the activity to avoid confusion when Fathom automatically converts $\frac{\text{ft}}{\text{mph}}$ to seconds.

Factored Form Quadratic—Gravity

A football is kicked from a height of 2 ft and hits the ground 3.5 s later.

Q1 How high do you think the ball rises before beginning to fall?

Q2 What factors might determine that height?

Any object rising and falling due to gravity can be modeled with the quadratic function $y = ax^2 + bx + c$, where y is the height in feet and x is the time in seconds since the object was launched.

INVESTIGATE

You can set up a graph to show your view of the football field.

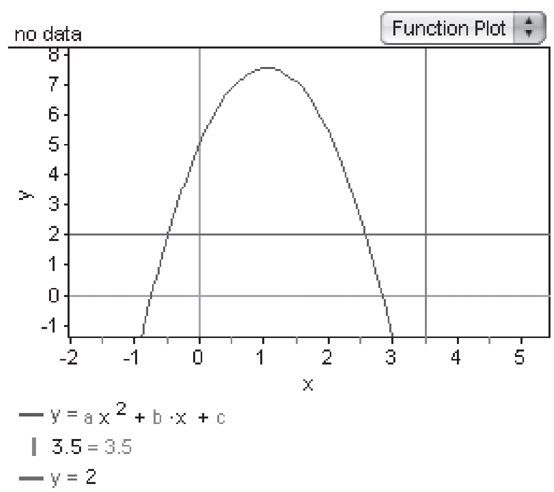
1. In a new Fathom document, drag from the shelf sliders for the coefficients a, b, and c. Drag a graph from the shelf. In the upper right corner of the graph, select **Function Plot** in the drop-down menu.

Make sure you click the multiplication sign between a and x^2 and between b and x.

2. To begin graphing the relationship between time and the height of the football, choose **Plot Function** from the **Graph** menu. Enter as an expression the quadratic function ax² + bx + c.

It might help to **Plot Value** 3.5 and **Plot Function** $y = 2$ as guides.

3. To make the parabola fit the data, adjust the values of a, b, and c until the graph represents a ball that is 2 ft high at time 0 s and 0 ft high at time 3.5 s. To make the parabola open downward, use negative values for a.



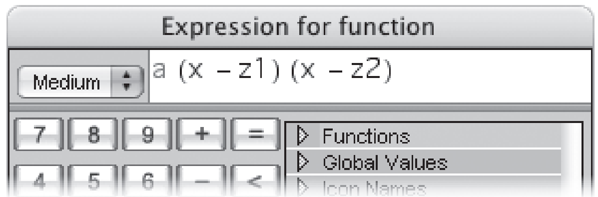
Q3 What values did you find for a, b, and c? What quadratic expression represents the height of the ball over time?

Q4 For your quadratic expression, how high does the ball rise?

Q5 On Earth, the value of a is -16. If you set a to -16, what values of the other sliders make the height 2 ft at 0 s and 0 ft at 3.5 s?

Q6 How high does the ball rise with these settings?

4. The *factored form* of the quadratic function is $y = a(x - z_1)(x - z_2)$. Drag sliders from the shelf for z_1 and z_2—for names, you might enter z1 and z2. Replace the equation you've graphed with one in this factored form.



Q7 Still using $a = -16$, what values for z_1 and z_2 give a graph that passes through (0, 2) and (3.5, 0)?

Q8 How do the values of z_1 and z_2 relate to the graph?

Q9 Why are z_1 and z_2 called the *zeros* of the function?

EXPLORE MORE

Look at how the sum of the two zeros relates to *a* and *b* and how the product of the zeros relates to *a* and *c*.

1. Go back to graphing $ax^2 + bx + c$. Experiment with changing the value of *a*. Keep track of the values of *a, b*, and *c* and the zeros of each function. How do the zeros relate to the values of *a*, *b*, and *c*?
2. Research how the values of *a* and *b* relate to kicking a football. If a ball kicked on Earth from a height of 2 ft hits the ground in 3.5 s, what was its initial upward velocity and how high did it go?

Factored Form Quadratic—Gravity

Activity Notes

Objective: Students will see relationships among the coefficients of a quadratic equation in general form, the factored form of the equation, and the function's zeros (the x-intercepts of its graph). Students are given the factored form and, by using sliders, discover how this form relates to the graph.

Student Audience: Algebra 1, Algebra 2

Activity Time: 25–35 minutes

Setting: Paired/Individual Activity

Mathematics Prerequisites: Students can read a function graph and locate x-intercepts**.**

Fathom Prerequisites: Students can use sliders, plot a function referring to a slider, and trace a function graph.

Fathom Skills: Students learn how to make a function plot.

Notes: As you talk with students, don't let them confuse the path of the football, also a parabola, with the function they graph in step 2—a function that relates the height of the football to time, not height and horizontal distance traveled. A wide variety of answers are possible on Q3 and Q4. You might ask students to check their own answers to make sure that with $c = 2$, $b = -3.5a - 0.57$. If students ask why, suggest that the quadratic formula would help them verify the relationship between a and b. As students are working on Q7, suggest that they enlarge the graph and ensure that the scale for z_1 is set to measure in thousandths. During a class discussion after the activity, or as students share their answers, make sure they can explain the answers to Q8 and Q9. A function's zeros are the x-intercepts of its graph, or the solutions to its equation.

Q1 Answers will vary considerably. You need not comment on them.

Q2 The primary influences are the force of gravity and the initial upward velocity of the ball when kicked. The angle of the kick affects the upward speed. Air resistance is another factor, which is ignored in this model.

INVESTIGATE

Q3 The many correct answers each have $c = 2$ and a negative. Theoretically, because one of the zeros of the function is at $x = 3.5$ for $c = 2$, $3.5 = \frac{-b \pm \sqrt{b^2 - 8a}}{2a}$. Therefore, b will be approximately equal to $-3.5a - 0.57$. For example, $-x^2 + 2.93x + 2$.

Q4 Answers depend on the equation from Q3. The maximum height is $c - \frac{b^2}{4a}.$ For the example from Q3, the y-coordinate of the vertex is 4.15.

Q5 $b \approx 55.4$, $c = 2$

Q6 About 50 ft

Q7 The z-values are about -0.036 and 3.5.

Q8 The z-values are the x-intercepts of the graph.

Q9 The zeros of a function are the values at which the function is 0.

EXPLORE MORE

1. Students can keep track of their data in a Fathom table, letting Fathom calculate the sum and product. The sum of the zeros is $-\frac{b}{a}$; their product is $\frac{c}{a}$.
2. The initial upward velocity of the ball is b; c is the ball's initial height; a is acceleration toward Earth due to gravity. The ball kicked on Earth would have initial upward velocity of 55.4 $\frac{\text{ft}}{\text{s}}$ and would reach a height of about 50 ft.

Vertex Form Quadratic—Protecting Wildflowers

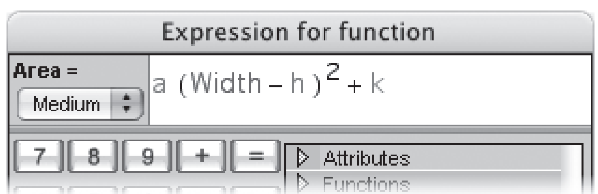
The school ecology club has permission to fence in a region along a riverbank to protect some endangered wildflowers that grow there. The club has enough money to buy 220 feet of fencing. It decides to enclose a rectangular space. The fence will form three sides of the rectangle, and the riverbank will form the fourth side.

Rather than trample down wildflowers, the club makes some rectangles along the side of the school to determine which has the largest area.

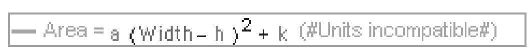
Q1 How large an area do you think can be fenced on three sides using 220 ft of fence?

INVESTIGATE

1. Open the document **Wild flowers.ftm** to see the data from the club's experimentation. Create a scatter plot of (*Width*, *Area*).
2. The data appear parabolic. Because you want to find a vertex of the parabola, it would be useful to use the vertex form: $Area = a(Width - h)^2 + k$. Create sliders for a, h, and k, and enter the function.



You get an error message that the units are incompatible.



3. To eliminate that message, you'll need to put units on each slider. Decide which sliders use feet and square feet.

Q2 What are the units for each slider?

4. Experiment with the sliders until you have a good fit for the data.

Q3 What are the values of the sliders?

Q4 How do these values relate to the problem?

Q5 How do these values relate to the graph?

Q6 Why is this form of the equation called the *vertex form*?

Q7 What are the dimensions of the largest rectangle?

EXPLORE MORE

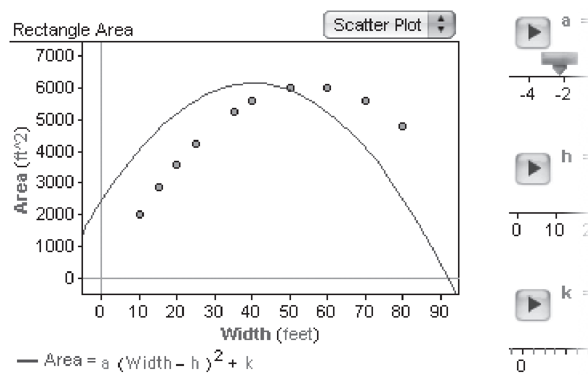
1. What if the rectangle is not along the riverbank but is enclosed only by fence? What length and width will give the maximum area?
2. One club member suggests that they would get more area if they used a trapezoid with 45° base angles instead of a rectangle. Use the data from the Trapezoid Area collection to discover whether this is correct.
3. Could forming the fence into a different nonrectangular shape enclose more area?

Vertex Form Quadratic—Protecting Wildflowers Exploration

The school ecology club has permission to fence in a region along a riverbank to protect some endangered wildflowers that grow there. The club has enough money to buy 220 ft of fencing. It decides to enclose a rectangular space. The fence will form three sides of the rectangle, and the riverbank will form the fourth side.

Rather than trample down wildflowers, the club makes some rectangles along the side of the school to determine which has the largest area. To see the data from the club's experimentation, open the document **Wildflowers.ftm.**

To model the data, it might be easy to start with the vertex of the parabola; you can use a function in vertex form: $Area = a(Width - h)^2 + k$, where the vertex is at (h, k). Use sliders for a, h, and k to find a function that fits the data and determine the dimensions of the largest rectangle.



EXPLORE MORE

1. What if the rectangle is not along the riverbank but is enclosed only by fence? What length and width will give the maximum area?
2. One club member suggests that they would get more area if they used a trapezoid with 45° base angles instead of a rectangle. Use the data from the Trapezoid Area collection to discover whether this is correct.
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Vertex Form Quadratic—Protecting Wildflowers Activity Notes

Objective: Students will create a strategy for modeling quadratic data based on the vertex-dilation equation. Fathom's sliders and units are used to discover the roles of the values used in the vertex form of the quadratic.

Student Audience: Algebra 1, Algebra 2

Activity Time: 30–45 minutes

Setting: Paired/Individual Activity, Exploration, or Whole-Class Presentation (use **Wildflowers.ftm** for any setting)

Mathematics Prerequisites: Students understand the graph of a function.

Fathom Prerequisites: Students can use sliders, create a scatter plot, and plot a function based on slider values.

Notes: Help students who are struggling to find the units for the sliders by asking about units when numbers are added or subtracted. Suggest they write the equation and then check that the units are the same on both sides of the equal sign. The open-ended version of the Exploration allows students to build on Fathom skills developed earlier. Some students might benefit from showing a residual plot as well. As students report values for h and k in Q3, check that they include units. If you can give students more than 25 minutes for this activity, many will be able to complete the Explore More questions. You might ask students who complete those additional questions to share their results with the class.

For a Presentation: Encourage discussion of Q2. You'll also want to present the Explore More questions.

Q1 Answers will vary.

INVESTIGATE

Q2 The a slider needs no units, the h slider should be in feet, and the k slider should be in square feet.

Q3 $a = -2$, $h = 55$ ft, $k = 6050$ ft^2

Q4 Values h and k give the desired width and maximum area. Value a does not directly relate to the problem.

Q5 Values h and k give the position of the parabola's vertex. Value a relates to the proportions of the parabola.

Q6 It is called the *vertex form* because the coordinates of the vertex are part of the equation.

Q7 The theoretical maximum area occurs when the width is one-quarter of the total 220 ft and the length is twice that: 55 ft $\cdot$ 110 ft $=$ 6050 ft^2.

EXPLORE MORE

1. The maximum area of a rectangle with a fixed perimeter occurs when the rectangle is a square. Students can use this fact to solve the original problem as well: Imagine reflecting the rectangle across the riverbank to have a rectangle with twice as much fencing (440 ft). Form that into a square, so each side has one-quarter of 440 ft. On the dry side of the riverbank, each width is half that long, or one-eighth of 440 ft, and the length is one-quarter of 440 ft.
2. A trapezoid will give greater area. The maximum area has the approximate dimensions of width 60 ft and bases of 50 ft and 170 ft, for an area of 6600 ft^2.
3. Regular polygons with the same perimeter will have more area if they have more edges. Of all figures with the same perimeter, the circle has the most area. Against a riverbank, the semicircle has more area than any other figure with the same perimeter.

EXTENSION

The first sentence of Explore More 3, known as the *Isoperimetric Theorem*, was known by the ancient Greeks. Suggest that students research the history of this theorem and its proof.

The Quadratic Formula — Exploration

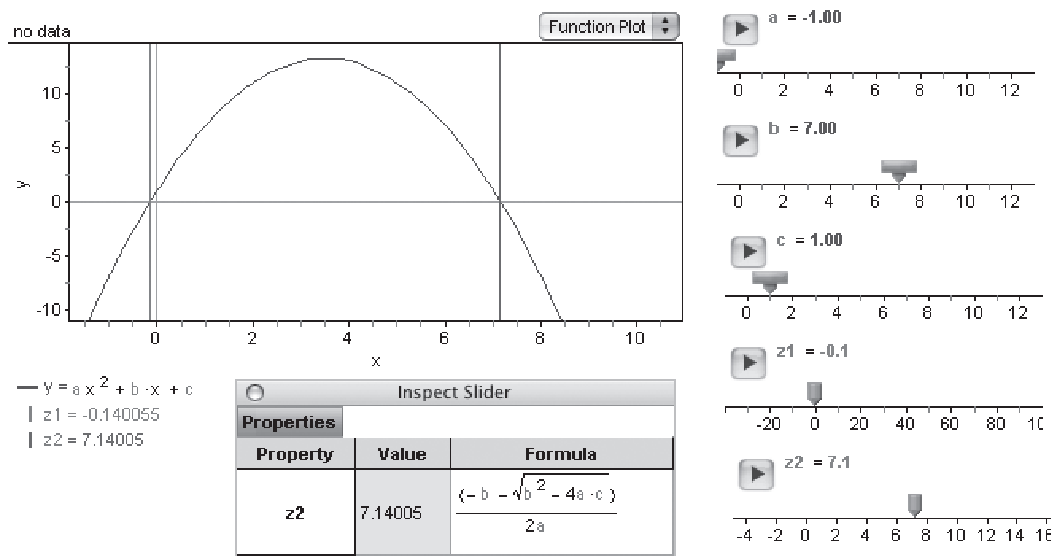
The quadratic formula says that you can calculate the zeros of the quadratic function $f(x) = ax^2 + bx + c$ by finding the two values of $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

CREATE A MODEL

In a new Fathom document, drag down sliders for the coefficients a, b, and c and make a function plot of the formula $ax^2 + bx + c$. Also drag down sliders for zeros z_1 and z_2 and give the sliders formulas for the zeros, as described by the quadratic formula.

INVESTIGATE

Explore what happens to the graph and the zeros as the values of a, b, and c change. What can you say about the graph when z_1 and z_2 have values and when they have domain errors? Explain.



The Quadratic Formula

Objective: Students will explore the roles of the coefficients of a quadratic function in the locations of the zeros, as given by the quadratic formula, and in the location of the *x*-intercepts of the function's graph.

Student Audience: Algebra 1, Algebra 2

Activity Time: 25–35 minutes

Setting: Paired/Individual Exploration

Mathematics Prerequisites: Students understand the graph of a quadratic, associating zeros of a function with the *x*-intercepts of the function's graph.

Fathom Prerequisites: Students can use sliders, make a function plot with a formula based on sliders, and assign formulas to sliders.

Notes: This open-ended exploration allows students to see how the values of the zeros of a quadratic function, as given by the quadratic formula, relate to the location of the parabolic graph. In particular, you want them to see that the zeros become undefined, with Fathom indicating #Domain error#, when the graph has no *x*-intercepts. Patterns seen in Fathom can motivate the question, How do *a, b,* and *c* relate to each other when there are zeros and when there aren't? The question can be answered algebraically by considering the discriminant, $b^2 - 4ac$.

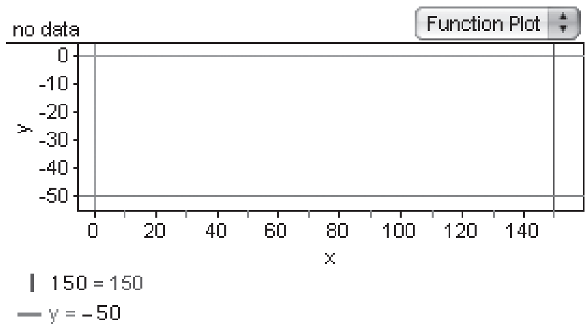
Parabola—Solar Oven

You want to build a parabolic solar oven to be fit into a frame. Your design is to have a dish 50 cm deep and 150 cm across at the level of the frame. You need a formula for the dish to give you the measurements you need to construct the oven.

INVESTIGATE

On the new graph, change Empty Plot to Function Plot. Then choose **Graph | Plot Value** and **Graph | Plot Function** to draw lines at $x = 150$ and $y = -50$.

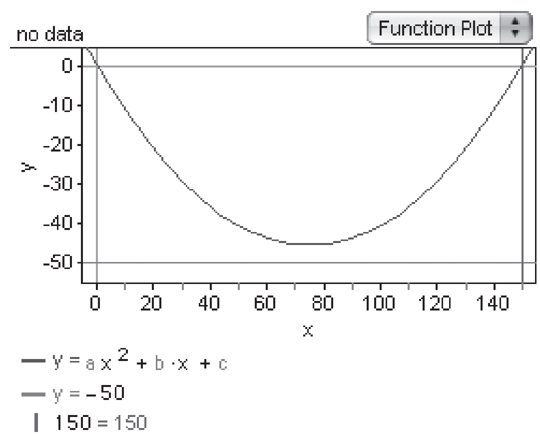
1. Open a new Fathom document and drag down a graph for a function plot. Use the dish measurements to help set the window. You might plot values to show the dimensions of the dish.



2. Because the dish design is parabolic, its formula will be $y = ax^2 + bx + c$. Drag down sliders for the coefficients a, b, and c.

Adjust the slider scales so that a, b, and c can take on values close to zero.

3. Plot the function with formula $ax^2 + bx + c$ and adjust the sliders to place the curve in the box. (Don't worry about a perfect fit just yet.)



The quadratic formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, will give you the location of the x-intercepts of any quadratic curve of the form $y = ax^2 + bx + c$.

Q1 What are the approximate x-intercepts of your current graph, and what are the x-intercepts of the curve you want?

Another way to write the quadratic formula is as two fractions: $\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. This form will help you fit your graph in the box.

Double-click on the slider's thumb to open its inspector and enter the formula.

4. Create a new slider, enter the name V1, and give it a formula of $\frac{-b}{2a}$. Then, with the graph selected, choose **Graph | Plot Value** to draw a vertical line on your graph at *V1*.

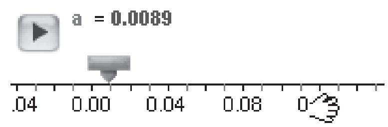
Q2 Where is this line on your parabola? (Check your hypothesis by moving the sliders and watching the line.)

Q3 Where will this line need to be on the finished parabola?

5. Set the expression $\frac{-b}{2a}$ equal to your answer to Q3 and solve for *b*. Enter this formula for slider *b*.

Q4 As you slide the value of *a*, how does the graph change?

Q5 What values of *a*, *b*, and *c* give a graph that best fits the box? You might need to adjust the scales to look at small intervals.



If you get the error message #Units incompatible#, you may have forgotten to click on the multiplication sign between 4, *a*, and *c* or between 2 and *a*.

6. Create two more sliders, using the values of *a*, *b*, *c* and the formulas $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$.

Q6 Aided by the values of the new sliders, can you improve your model?

Q7 To help construct the parabola, how high off the bottom of the oven are the points that are 40 cm from the center horizontally?

EXPLORE MORE

Experiment with the *a, b,* and *c* sliders until the values of the quadratic formula sliders give an error message. What is different about this graph, as compared with graphs that don't give error messages?

Parabola—Solar Oven

Objective: Students will use a quadratic equation to model a solar oven and will discover some of the relationships between the quadratic formula and the graph.

Student Audience: Algebra 1, Algebra 2

Activity Time: 25–35 minutes

Setting: Paired/Individual Activity or Whole-Class Presentation

Mathematics Prerequisites: Students understand the graph of a quadratic and associate zeros of a function with the *x*-intercepts of the function's graph.

Fathom Prerequisites: Students can create sliders, make a function plot with a formula based on sliders, and assign formulas to sliders.

Notes: The margin note for step 3 will help students who are having trouble getting the curve into the box. As you listen to students working, look for an understanding that $\frac{-b}{2a}$ gives the horizontal component of the vertex and that the symmetry of this curve means the two intercepts must be equidistant from this value. As students work on Q6 ask, How will the values of the new sliders help? The distance $\frac{\sqrt{b^2 - 4ac}}{2a}$ is both added and subtracted from the center to find the intercepts. For students familiar with function notation, you might ask them to use function notation as they write their answer: $f(35) = f(115) \approx 35.8$.

For a Presentation: Set the values of *a* and *c* near 0 and the value of *b* near -1 to get the curve to roughly fit the box. Ask several students to describe what they see at Q4. As the sliders are created in step 6, ask what these slider values will be when the curve fits the box. After tracing the curve to find the answer to Q7, you might extend the question to other numbers, such as 30 cm from the center or 20 cm from the top.

INVESTIGATE

1. The following answers assume that the dish will be below the *x*-axis and to the right of the *y*-axis. The screen captures in the activity suggests this placement. However, there are other valid ways to begin the activity.

Q1 The intercepts of student curves will vary. The desired intercepts are 0 and 150 cm.

Q2 It passes through the vertex of the parabola.

Q3 The *x*-coordinate of the vertex should be at 75 cm.

5. $75 = \frac{-b}{2a}$, $b = -75 \cdot 2a = -150a$

Q4 The *x*-value of the vertex is always at 75 cm. The *y*-value of the vertex increases and decreases.

Q5 Best answer is $a = \frac{2}{225}$, or about 0.00889; $b = -\frac{4}{3}$, or about -1.333; and $c = 0$.

Q6 Same as Q5

Q7 About 35.77 cm below the top, or about 14.22 cm above the bottom

EXPLORE MORE

A parabola with errors instead of values is a parabola that does not cross the horizontal axis.

Binomial Products—Sales and Profits

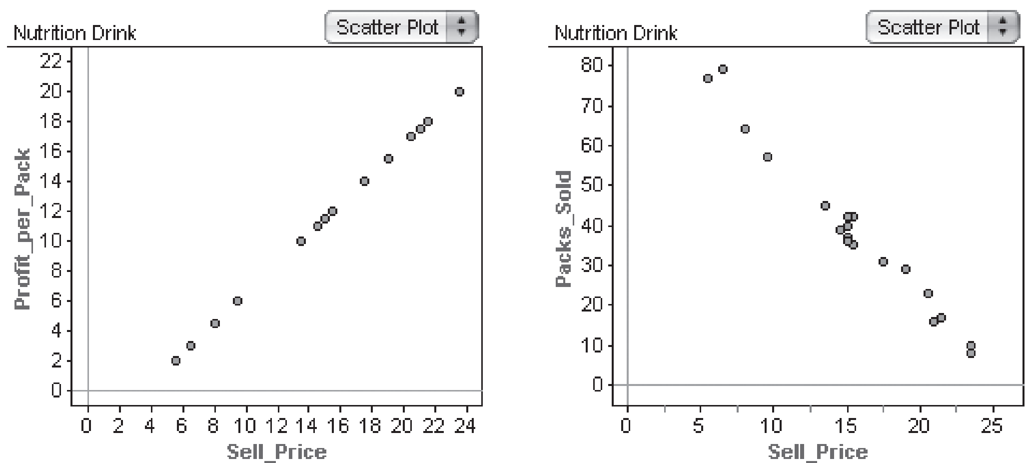
You have developed a great-tasting nutrition drink. You sell it in 12-packs to 20 retail markets in your area. Some of the discount stores resell the 12-packs at a low price so as to sell a large number of packs. Some health clubs sell drinks individually at a high price and sell only a few packs. You have decided to sell your own product at a local festival, but you need to choose a price.

Q1 Is it better to sell many drinks at a low price or a few at a high price? Explain your ideas.

Test your opinions by collecting data on last month's sales at each outlet.

INVESTIGATE

1. Open the Fathom document **Sales.ftm**. You will find a case table of the selling price per pack from each outlet, the profit they made on each pack, and the total sales for the previous month. You can start your research by looking for any patterns in these values. Create scatter plots for *Sell_Price* versus *Profit_per_Pack* and *Sell_Price* versus *Packs_Sold*.



2. Find the best line of fit modeling each of these graphs.

Q2 What model did you use for the first graph? What can you learn from its intercepts with the axes?

Q3 Give the model for the second graph and explain what the slope in this model tells you.

Q4 How would you calculate how much money Albert's Market made from this product last month?

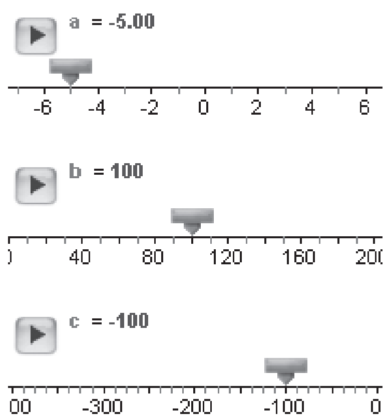
Nutrition Drink

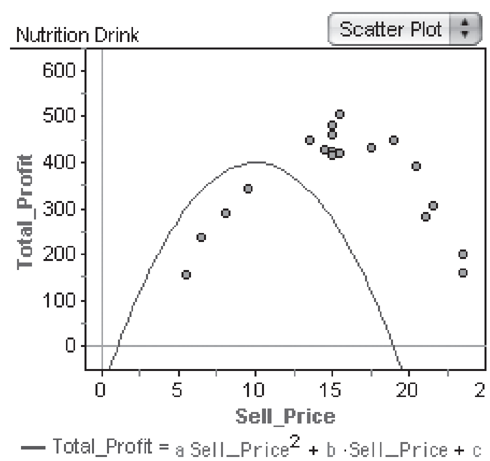
	Outlet	Sell_Price	Profit_per_Pack	Packs_Sold
1	Albert's Market	13.5	10	45

3. What you want to know is which stores made the most money. Add a new attribute in the case table with a name like *Total_Profit* and enter a formula to calculate this value.

Q5 What formula did you use? Which outlet made the greatest profit?

4. Because the best price according to the model may not be one of the prices any outlet charged, you will want to look for a formula. Create a third graph to study how profit relates to the selling price. Use sliders to help you find a model to fit these data.





Q6 What model did you find to fit these data? According to that model, what price should you charge at the festival, and what profit will you receive?

Now that you have solved the problem one way, you wonder whether using algebra can give you a solution without using sliders. You decide to compare the three graphs.

Q7 What are the horizontal intercepts of the three models? Explain any patterns you see.

Q8 Multiply the right sides of your answers to Q2 and Q3. Explain any patterns you see.

Q9 How could you have found the model for *Sell_Price* as a function of *Total_Profit* without sliders? What solution would you have gotten?

EXPLORE MORE

1. You have decided to go into the vegetable-growing business. You plant carrots. When these carrots are young, they are very sweet, but they get less tasty as they continue to grow in size. Each day you leave them in the field, they increase in size. Create a case table for the collection Carrots (scroll down in the Fathom document **Sales.ftm**) and use what you have learned to find the best day to harvest your crop for the highest profit. The values given are based on previous years and are only good approximations of this year's crop. A function for *Profit* in terms of *Time* will help you find the best theoretical values.
2. Explore the Walleye data (also in **Sales.ftm**) to determine for the fishery the best age at which to sell fish to make the highest profit.

Objective: Students will explore how linear models can give information, both graphical and symbolic, about the quadratic model that is their product.

Student Audience: Algebra 1, Algebra 2

Activity Time: 30–40 minutes

Setting: Paired/Individual Activity or Whole-Class Presentation (use **Sales.ftm** for either setting)

Mathematics Prerequisites: Students can multiply binomials.

Fathom Prerequisites: Students can create a scatter plot, use movable lines, add attributes and edit formulas, use sliders, add function plots to a graph, and trace functions.

Notes: Step 2, Q2, Q3, and Q6 give students a chance to find a line (or curve) of fit and interpret the meaning of each function's terms for this problem situation. Students are gaining experience applying the process of finding a mathematical model to fit a situation, solving the model, and then interpreting the result back into the problem situation. Q7 and Q8 give students further experience with looking for patterns—doing mathematics.

For a Presentation: Ask several students to interpret the meaning of the constants and the coefficients in the lines of fit. Before you create the graph in step 4, ask students what shape they think the points on the scatter plot will have.

Q1 Answers will vary widely. You need not reach consensus at this time.

INVESTIGATE

2. Students may use movable lines, one of the built-in regressions, or the equation through two representative points. If they write the equation in point-slope form, encourage them to change it to intercept form to facilitate later calculations.

Q2 $Profit\_per\_Pack = -3.5 + Sell\_Price$, exact values may differ slightly. The vertical intercept is the per-pack wholesale cost to the retailer. Each item (pack) costs each store \$3.50. The horizontal intercept gives sales that would yield a profit of 0. Selling packs at \$3.50 would return no profit.

Q3 $Packs\_sold = 96 - 3.72Sell\_Price$, exact values may differ slightly. The slope is the rate at which the number of sales decreases as the price increases. The retailer gets 3.72 fewer sales for each dollar increase in price.

Q4 Multiply \$10 per item times 45 items sold to get \$450 profit.

Q5 $Total\_Profit = Profit\_per\_Pack \cdot Packs\_sold$. Don's Beverage #1 and #3 made a profit of \$504 for the month.

Q6 $Total\_Profit = -3.72Sell\_Price^2 + 109Sell\_Price - 336$, exact values may differ slightly. The best price is about \$14.65 per pack (or \$1.22 each), with the expectation of selling about 40 packs for a profit near \$462. To avoid dealing with pennies, \$1.25 each is a good price for the festival.

Q7 Graph 1: intercept at $x = 3.5$ (representing zero profit). Graph 2: intercept at 25.8 (representing zero sales). Graph 3: intercepts at 3.5 and 25.8 (representing zero profit for either of these reasons). The zeros of the profit function are the zeros of its factors.

Q8 The product should be something like that in Q6. The profit function is the product of the other two functions.

Q9 The product of the two linear expressions is a quadratic whose zeros are those of the linear functions. The problem could have been solved by graphing the product of the two linear functions, to get a price of \$14.65 with a profit near \$462.

EXPLORE MORE

1. A good model to fit ($Time, Weight$) is $Weight = -180 + 5.23Time$. A good model to fit ($Time, Price$) is $Price = 3.4 - 0.0298Time$. The profit model is the product of these two: $Profit = -0.156Time^2 + 23.15Time - 612$. The best time to sell is at 74 days, making a profit of \$246.85.

2. A good model for ($Age, Weight$) is $Weight = -0.75 + 0.212Age$. A good model for ($Age, UnitPrice$) is $UnitPrice = 17 - 0.329Age$. The profit model is the product of these two: $Profit = -0.070Age^2 + 3.85Age - 12.8$. The best age to sell is at 27.5 days, yielding a profit of \$40.14.

Binomial Expansion—Flipping Coins

After you flip a coin, either the heads side or the tails side shows.

Q1 If you flip three coins, do you think it's more likely that you will see two heads or three heads?

Fathom can help you simulate the flipping of coins, so you can experiment with many flips without taking much time or wearing out your fingers.

INVESTIGATE

One way to simulate flipping coins is to set up a collection with two members, representing the outcomes of heads and tails. To flip three coins, take a random sample collection three times.

1. Open the Fathom document **Coin Flip.ftm**. You will see a Coin collection with an enlarged window, allowing you to see its two members: one Head and one Tail. The repeated draws from the Coin collection are in a different collection, Flips. At this point, the Flips collection contains the results of three coin flips. Click on **Sample More Cases** in the Flips collection to see the results of flipping three coins again.

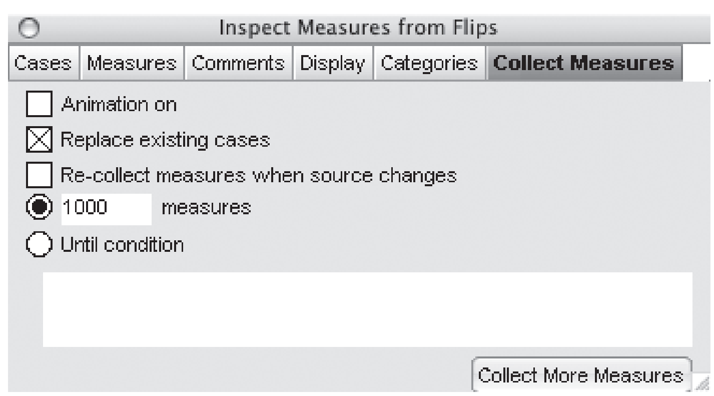
Q2 Keep track of the results as you simulate flipping the three coins eight times. How many of those times do two coins show heads? How many of those times do three coins show heads?

Q3 Answer Q2 for another eight simulations.

The document contains a third collection, which will keep track of the results for you. The results are called *measures,* and this collection is named Measures from Flips.

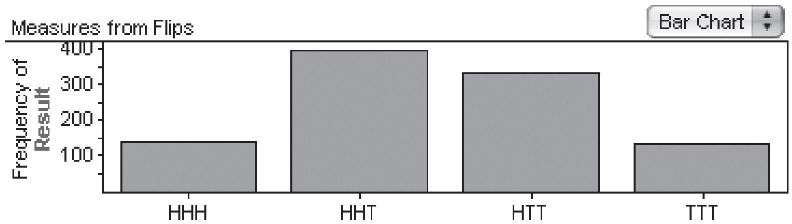
To speed up the simulation, make sure animation is not on.

2. Open the inspector for Measures from Flips and use it to collect 1000 measures.



This graph was created by dragging the *Result* attribute from the **Cases** panel of the Measures from Flips inspector to the horizontal axis of a new graph.

3. Create graphs and summary tables to use this information to help you answer the questions.



Measures from Flips

Result	HHH	112
	HHT	387
	HTT	386
	TTT	115
Column Summary		1000

S1 = count ( )

To create this summary chart from a different sample of 1000 flips of three coins, highlight the measures collection and drag a summary table from the object shelf. Open the measures collection inspector, click the **Cases** tab, and drag and drop *Result* on the down arrow in the summary table.

Q4 How likely is three heads? Two heads and one tail? One head and two tails? Three tails?

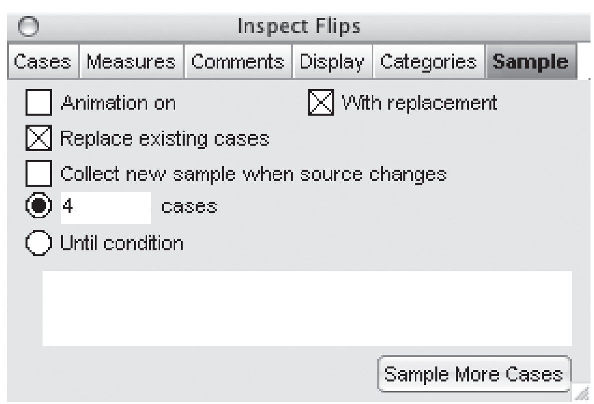
Q5 To find the connection between coin flipping and algebra, multiply $(H + T)(H + T)$, and then multiply the result by $(H + T)$ again.

Q6 Combine the like terms after you have multiplied. What is $(H + T)^2$? How do the coefficients relate to the coin flips?

You have seen the result for three flips. Does it apply to other numbers of flips?

4. To try for four flips, open the inspector for the Flips collection and change the number of cases to 4. Simulate flipping the four coins 16 times, recording the number of heads each time.

Make sure you open the inspector with the title Inspect Flips.



When the Flips collection is updated to simulate flipping four coins, Measures from Flips will measure the number of heads (and tails) in four coins.

Q7 What were your results? Repeat another 16 times. What did you get this time? Combine your results with those of classmates and find the average numbers for 16 flips.

Q8 Multiply $(H + T)$ by the expansion you found in Q5. Combine the like terms after you have multiplied. How do the coefficients relate to the coin flips?

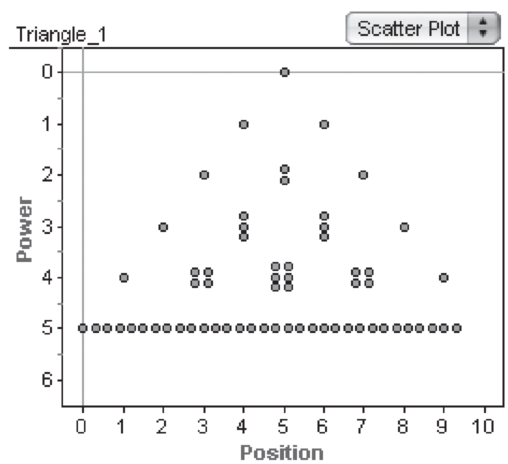
Q9 Calculate $(H + T)^5$ and guess at what fraction of the time flipping five coins will produce four heads.

5. To test your conjecture, adjust your simulation as you did in step 4 so that you are sampling five flips. Simulate 32 flips several times.

Q10 Use algebra to conjecture what fraction of the time flipping six coins will produce two heads. To test this pattern, simulate 64 flips several times.

EXPLORE MORE

Scroll down in the **Coin Flip.ftm** document to find a graph labeled Triangle_1. Drag the corner to enlarge the graph. The rows of the triangle have 1, 2, 4, 8, 16, and 32 points, respectively. All but the last row are arranged as the coefficients of a binomial expansion. Look for patterns and then arrange the 32 points in the last row to fit the pattern.



Extend the pattern further by writing numbers instead of arranging dots. Do you recognize this famous number triangle?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 _ _ _ _
1 6 _ _ _ _ _

Binomials Expansion—Flipping Coins — Activity Notes

Objective: Students will explore how patterns in binomial expansions relate to flipping coins.

Student Audience: Algebra 1, Algebra 2

Activity Time: 15–25 minutes

Setting: Paired/Individual Activity (use **Coin Flip.ftm**)

Mathematics Prerequisites: Students can multiply polynomials.

Fathom Prerequisites: Students have worked with collections.

Notes: As you introduce the activity or talk with students, help them understand two powerful tools of Fathom—the ability to collect samples and the ability to collect measures from those samples. Collecting samples involves cases selected at random from the original collection and assembled into a new collection. In this activity, the original collection is the two possible outcomes for a coin flip, and the sample collection is one flip of three coins. To see the trend in the long run, you take many samples. The more samples you take, the closer the result will be to the result in the long run.

You need a tool to keep track of the count of heads for each flip of three coins. A measure is a calculation based on the sample collection. For example, you can count the number of heads in each sample and save that information—in this case, it is saved in Measures from Flips. When Fathom collects measures, it re-collects the sample, calculates the formula created for each measure (counts the number of heads), and then builds a new collection with the results. The attributes in this new collection are the measures—here, count of heads—and each case represents a measure taken from a different random sample—here, one flip of three coins.

Ask students who finish early to take measures from 1000 simulations of flipping four coins, five coins, or more. Students who are interested might examine and explain the formulas for the measures, which are taken as the sample is collected and then sent to the measures collection.

During sharing, the class can look at and combine several results for Q4 by making a table in Fathom for class results and then using Fathom to combine those results to get another picture of the probability in the long run. Taking samples with the animation on can help students see what is happening—where the numbers are coming from. You might show sampling from coins five times with animation on. When collecting a large number of samples, however, turn off the animation.

Q1 Answers may vary substantially. Consensus will come through the activity.

INVESTIGATE

Q2 Answers will vary. The number of times two coins show heads will be about three times the number of times all three coins show heads.

Q3 Same as Q2

Q4 There will be approximately three times as many two heads and a tail or two tails and a head as there are three coins the same.

Q5 $HHH + 3HHT + 3HTT + TTT$, or $H^3 + 3H^2T + 3HT^2 + T^3$

Q6 The coefficients are roughly the same as the counts. Point out that the term $3H^2T$ means that two H's and one T are multiplied in three different ways when expanding the binomial; two heads and one tail can occur *HHT, HTH,* or *THH.*

Q7 For a large number of trials, the number of times various numbers of heads will appear are approximately 1, 4, 6, 4, 1. With only 16 trials, the actual results may vary so greatly from the theoretical result that the patterns may not appear. Combining data for the class will help reveal the pattern.

Q8 $HHHH + 4HHHT + 6HHTT + 4HTTT + TTTT$, or $H^4 + 4H^3T + 6H^2T^2 + 4HT^3 + T^4$

Q9 $H^5 + 5H^4T + 10H^3T^2 + 10H^2T^3 + 5HT^4 + T^5$. Four heads will appear about 5 times out of 32.

Q10 $H^6 + 6H^5T + 15H^4T^2 + 20H^3T^3 + 15H^2T^4 + 6HT^5 + T^6$. Two heads will appear about 15 times out of 64.

EXPLORE MORE

The bottom row of the dots should be arranged in groups of 1, 5, 10, 10, 5, 1. The pattern is Pascal's triangle; each number is the sum of the two numbers above it. The last row pictured is 1, 6, 15, 20, 15, 6, 1.

Common Factor—Acceleration

Acceleration due to gravity affects how fast objects fall, how projectiles travel, and how much force is needed to launch rockets. But acceleration differs around the world. Your goal in this activity is to find the value of that acceleration in a particular place.

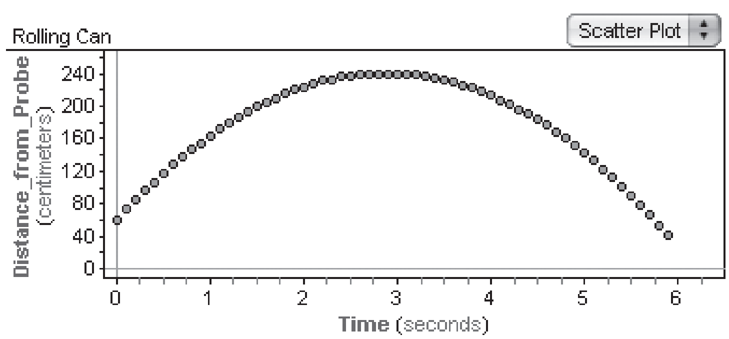
Because falling objects travel so fast that measurements are difficult, the famous scientist Galileo devised a way to find the acceleration due to gravity by using objects rolling on a ramp, or an incline. Here you'll be using some data collected on an incline.

INVESTIGATE

1. Open the Fathom document **Acceleration.ftm**. You will see a table of distance and time data from an experiment.

To collect these data, an incline was constructed by raising by 11 cm one end of a 2.44 m table. A probe for measuring distance and time was placed at the bottom. A can was placed on the incline above the probe and given a shove upward. The measuring device recorded the distance between the can and the probe at various times.

2. Create a scatter plot of *Distance_from_Prob*e versus *Time*.



Q1 What type of model would you use for these data? How far from the probe was the can when the data collection started?

3. You could find a model for these data with sliders, but here you'll use an approach based on factoring. If z is a zero of the function (represented on the graph by a horizontal intercept), then $(Time - z)$ is a factor of the function. To make this factor easy to use, adjust the graph so that 0 is a zero of the function. Create a new variable that measures the distance from the start of the data collection rather than the distance from the probe. Name this new attribute

something like *Distance_from_Start*, enter the formula shown here, and change the dependent variable on the graph.

Q2 How does this graph differ from the first graph? Explain.

Now, $(Time - 0)$, or simply *Time*, is a factor of the *Distance_from_Start* function. If you divide *Distance_from_Start* by *Time*, you should get linear data, which are easy to model. $\frac{Distance\_from\_Start}{Time}$ even has a meaning: It is average speed.

4. To get the linear data, create a new attribute called *Average_Speed*, give it the formula shown here, and make a scatter plot of *Average_Speed* versus *Time*.

Q3 Describe this graph.

Q4 What equation fits the data in this graph?

5. To see how this model can be used to find a quadratic model, start with the equation $d = at^2 + bt + c$ and subtract c from both sides. Then divide both sides by t.

Q5 What is the resulting equation, and how does this new equation relate to your graph in Q2 and its equation?

Q6 Using your answer to Q4 and the factor *Time*, find a quadratic equation to model the original data. Check your equation by graphing it on a scatter plot of *Distance_from_Probe* versus *Time*.

Q7 The value of a in $d = at^2 + bt + c$ is the acceleration of the can along the incline. Because the incline rises 11 cm over 244 cm of length, the acceleration a is related to the acceleration due to gravity, which we'll call g, by the formula $a = \frac{1}{2}g\left(\frac{11}{244}\right)$. Solve this equation to find the acceleration due to gravity where the experiment was performed.

Q8 Explain how factoring helped you to find a model for the quadratic.

EXPLORE MORE

Many snowplows have large cone-shaped bins filled with sand and salt to spread on snow-covered roads. It is quite time-consuming to refill the bins, but it is also time-consuming to have to stop in the middle of a route to return because you have run out. You wish to build a model that the road crews could use to estimate how far they can travel before they are out of sand/salt. In the collection called Salt Truck, the road crews have provided you with 20 data points collected where they have recorded the height of the sand and distance the plow went before it was empty. Describe the steps taken to model these data with a function in the same way you proceeded in this activity. What is the function?

Common Factor—Acceleration

Activity Notes

Objective: Students will use factoring to find a quadratic model of some real-world data involving motion on an incline.

Student Audience: Algebra 1, Algebra 2

Activity Time: 15–25 minutes

Setting: Paired/Individual Activity (use **Acceleration.ftm**)

Mathematics Prerequisites: Students can solve equations.

Fathom Prerequisites: Students can create a scatter plot, add attributes and edit their formulas, use movable lines to find a line of fit, and add function plots.

Notes: As you talk with students, encourage them to keep the units in mind; distances in centimeters, speeds in centimeters per second, acceleration in centimeters per second squared. The ratio of distances is unitless; it results from dividing centimeters by centimeters. As students work on Q3, they might note that the points near the y-axis, where both the time and the distance are small, show the greatest error. They can ignore those points when they fit the graph. The text before step 4 is essential for students to understand; they know one factor of the *Distance_from_Start* quadratic function; by dividing by *Time*, they will find the other factor. Ask several students to share their answer to Q6 and the explanation they wrote for Q8.

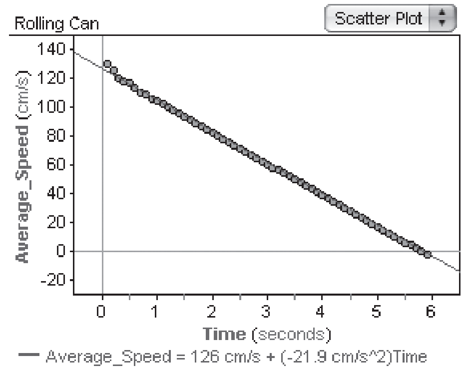
INVESTIGATE

Q1 These data points seem parabolic, so a quadratic model should be best. The first data value (at time = 0) is 60 cm. The can was 60 cm from the probe when the data collection started.

Q2 It has the same shape, but it is 60 units lower and now passes through the origin.

Q3 This graph shows a linear relationship.

Q4 Answers may vary, but they'll be close to the least squares line of
Average_Speed = −22.1 cm/s² *Time* + 126 cm/s



Q5 $d = at^2 + bt + c$

$d - c = at^2 + bt$

$d - c = t(at + b)$

$\frac{d-c}{t} = at + b$

The ratio $\frac{d-c}{t}$ gives average speed, so the line on the graph of *Average_Speed* versus *Time* has a slope of a and a vertical intercept of b. The equation for this line will be one factor in the quadratic function that fits the *Distance_from_Start* function.

Q6 $-22.1t^2 + 126t + 60$, or
$-22.1\frac{\text{cm}}{\text{s}^2}\mathit{Time}^2 + 126\frac{\text{cm}}{\text{s}}\mathit{Time} + 60 \text{ cm}$

Q7 $a = \frac{1}{2}g\left(\frac{11}{244}\right)$

$-22.1 = \frac{1}{2}g\left(\frac{11}{244}\right)$

$-22.1 = \frac{11}{488}g$

$-980\frac{\text{cm}}{\text{s}^2} \approx g$

Q8 Many observations are possible here. A remark that the two expressions $at^2 + bt + c$ and $t(at + b) + c$ are the same would show good understanding.

EXPLORE MORE

The height is 0 at about 11.5 mi, so set up a new variable at which *Height* will be 0. Divide the new variable, *Reduced_Height,* by *Height* to get *Average* and find a line of fit for the data (*Height, Average*).

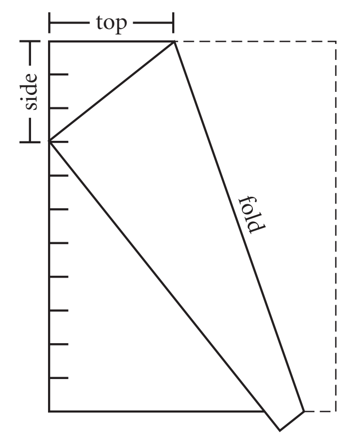
$$\mathit{Reduced\_Miles} = \mathit{Miles} - 11.5$$
$$\mathit{Average} = \frac{\mathit{Reduced\_Miles}}{\mathit{Height}}$$

The least-squares line through the data for (*Height, Average)* is *Average* = 4.90*Height* − 10.5. So one approximate model is *Miles* = 4.90*Height*² − 10.3*Height* + 11.5.

Polynomial Factoring—Maximum Area

Mathematical analysts in business and industry collect data and create models to find maximums, such as the maximum yield or the maximum profit, and minimums, such as the minimum waste or the minimum cost. Simple problems like folding a sheet of paper to find the largest triangle are used to train analysts for their work.

On a sheet of 8.5-by-11-inch paper, mark each inch from top to bottom along the left 11-inch edge of the sheet. Fold the upper right corner to one of the marks and crease the paper. There is now a right triangle of a single thickness in the upper left corner of the page, above the part of the edge that is folded. The two legs of the triangle are along the side and the top of the paper.



Q1 Which mark do you believe will result in the triangle with the largest area?

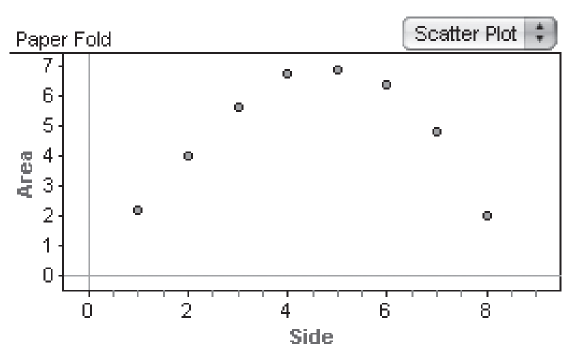
INVESTIGATE

The analyst must find the exact position of the fold to give the largest area in that triangle.

1. Open a new Fathom document. Drag down a case table and create the attributes *Side* and *Top*. Record in the table the lengths of the triangle's leg as you move the top right corner to marks along the left edge.

Q2 How many marks can you actually use? Explain.

2. Create a new attribute for *Area,* using the formula $0.5 \cdot Side \cdot Top$. Your goal is to find the exact position for the fold that makes the triangle the largest. To help you see the data, create a scatter plot of *Area* versus *Side.*



The graph looks somewhat quadratic. The graph of a quadratic function has symmetry, with the highest point halfway between the horizontal intercepts.

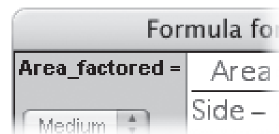
Q3 Think about how you gathered these data. Where should the horizontal intercepts be? That is, which values of *Side* would give you 0 area? What point is halfway between the two side lengths with no area?

Q4 Do you believe these data are actually quadratic? Why or why not?

The easiest type of model to find is linear. Often in statistics, you look for ways to change the data in order to "unbend" the curves, then you reverse the process to bend the line after you have found a model. This sequence is called *linearization.*

If z is a zero of a function, meaning the horizontal intercept of the graph, then $(x - z)$ is a factor of the function. Because you know two intercepts of this graph, you know two factors. You can create a data set of lower degree, and therefore one that is more linear, by dividing the data by one factor.

3. Create a new attribute called something like *Area_factored* and give it the formula of *Area* divided by one of the factors you know.

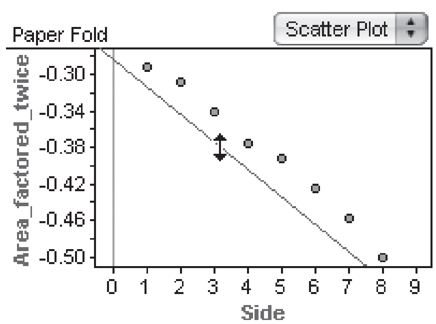


4. The original data will be quadratic if the values of *Area_factored* are linear. Make a scatter plot of *Area_factored* versus *Side.*

Q5 Is this graph linear or curved? Is it increasing, decreasing, or both?

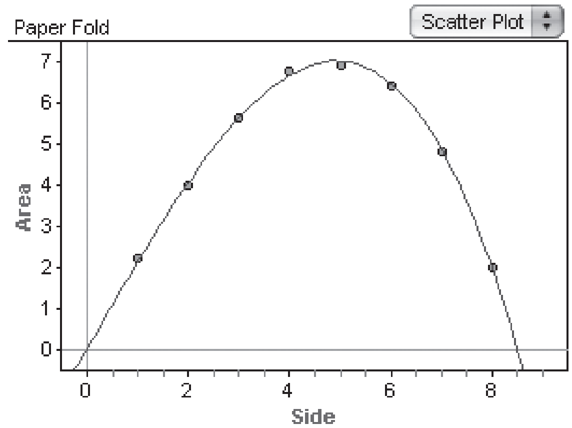
5. Because the data are not yet linear, divide *Area_factored* by the other factor, creating *Area_factored_twice.* Create a scatter plot of this new attribute versus *Side.*

Q6 Use a movable line to find a linear model for the data points in this graph.



6. To find the model you're seeking for area, work backward. Start with the equation you found in Q6 and multiply it by each of the factors you used to

make the data linear. Test this model by plotting it as a function on the scatter plot of *Area* versus *Side*.



Q7 What is your model for the area?

Q8 What does tracing the graph tell you about how to fold the paper to get a triangle of maximum area? According to your model, what is that area?

EXPLORE MORE

You found models for *Area_factored_twice* and for *Area*. How can you adjust the model for *Area_factored_twice* to get a model for *Area_factored*?

Objective: Students will use factoring as a part of a process of modeling a third-degree polynomial. Students will explore the relationships among intercepts, zeros, and factors as they maximize area in a paper-folding activity.

Student Audience: Algebra 1, Algebra 2

Activity Time: 20–35 minutes

Setting: Paired/Individual Activity or Whole-Class Presentation

Optional Document: Area.ftm

Mathematics Prerequisites: Students can solve equations.

Fathom Prerequisites: Students can create a data table with numeric and formula attributes, edit attribute formulas, use movable lines, and add function plots.

Notes: This activity can start with the collection of data using a sheet of 8.5-by-11−inch paper and a ruler, or you can save time and use the premeasured data in **Area.ftm.** You might start by demonstrating how to fold the paper and showing the location of the triangle that students need to measure. If your time is limited and you start with the data in **Area.ftm,** first demonstrate what is being measured. If students are confused, go back to the physical model, perhaps labeling the side and the top. As you visit working pairs, find one group who divided first by ($Side - 0$) and another that used ($Side - 8.5$). Ask both pairs to be prepared to share.

For a Presentation: If you only have access to one computer with presentation capability, you can still ask students to gather the data. Start a table in Fathom to enter each group's *Side* and *Top* measurements for each inch mark, then use the class average for the presentation. You might plot the value $x = 8.5$ and talk about Q4 and Q5.

Before the student running the computer shows the graphs in steps 4–6, ask what students expect to see. Ask the Explore More question.

Q1 Students will likely pick the 4 in. mark or 4.25 (halfway between 0 and 8.5). This is a good guess, but it is not so exact as the one they will derive later.

INVESTIGATE

Q2 At inch marks below 8 in., there is no triangle.

Q3 At 0 in. and at 8.5 in. If students have trouble, encourage them to think about a triangle with no area.

Q4 If the graph were symmetric, then the maximum would be at 4.25 in., but it is not. The data are probably not quadratic.

Q5 The graph is not linear. It will be decreasing if students divide by the factor ($Side - 0$), but it will be increasing if they divide by the factor ($Side - 8.5$).

Q6 Answers will vary, depending on the accuracy of students' original measurements. The model is something like $y = -0.03Side - 0.25$.

Q7 Using the above answer to Q6, $Area = (Side - 0)(Side - 8.5)(-0.03Side - 0.25)$.

Q8 Answers will vary. The maximum area occurs when *Side* is about 4.9 in., giving an area of more than 7 in.^{2}

EXPLORE MORE

$Area\_factored = Area\_factored\_twice \cdot Side$ or
$Area\_factored = Area\_factored\_twice \cdot (Side - 8.5)$,
depending on the last factor divided out